VIBRATIONS OF PLATES WITH CUT OUTS

By

SATISH CHANDRA AGRAWAL

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VIBRATIONS OF PLATES WITH CUT OUTS

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
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By
SATISH CHANDRA AGRAWAL

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CERTIFICATE

This is to certify that the thesis entitled 'Vibrations of plates with cutouts' by S.C. Agrawal is a record of work carried out under my supervision and has not been submitted elsewhere for a degree.

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POST GRADUATE OFFICE
This thesis has been approved
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SATISH CHANDRA AGRAWAL

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[redacted]

NOMENCLATURE

a	:	Plate length in y - direction
a'	:	Effective Plate length in y - direction
b	:	Plate length in x - direction
b'	:	Effective Plate length in x - direction
c	:	Frequency ratio parameter (ratio of frequency of plate to fundamental frequency of plate without cutout)
d	:	Length of cutout in y - direction
D	:	Flexural rigidity $(E h^3/12 (1 - \nu^2))$
e	:	Length of Cutout in x - direction
E	:	Young's Modulus
f	:	Natural frequency of plate cps
f_{fw}	:	Fundamental frequency of plate without cutout cps
h	:	Thickness of the plate
K	:	Nondimentional frequency parameter $(wa^2 \sqrt{\rho / D})$
ρ	:	Mass density per unit area
n	:	Constant for a particular clamping
r	:	Cutout size parameter (d/a , e/b)
s	:	Clamped length
x, y	:	Position coordinates
$\bar{Y}$	:	Distance of cutout center from plate center in y direction
z	:	Area ratio (ratio of cutout area to plate area)
ν	:	Poisson's Ratio
w	:	Natural frequency rad/sec

SYNOPSIS

of the
Dissertation on

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Natural frequencies and mode shapes of plates with cutouts have been investigated experimentally.

Acoustic excitation has been used to vibrate the plates. First three natural frequencies and the corresponding mode shapes of clamped - clamped rectangular plates with cutouts, for different aspect ratios, cutout sizes, eccentricity of the cutout and the cutout shapes, have been recorded.

Results are given in terms of non dimensional frequency parameter plotted against the cutout size parameter. The experimentally obtained results are also compared with available theoretical and experimental results. It is found that the fundamental frequency of the plate increases with cutout size. Shifting of the cutout towards the clamped edge reduces the fundamental frequency. For constant area ratio, the effect of cutout shape on fundamental frequency is not significant.

[redacted]

CHAPTER I

INTRODUCTION

1.1 General

Even though during the last few decades, the problem of vibrations of plates has been studied extensively, little attention has been paid to the vibrations of plates with cut - outs. Plates with cut - outs are extensively used in civil, naval and aeronautical structures. The examples are ship hull, aeroplane structures etc. The use of cut - outs becomes inevitable now-a-days because of modern architectural and industrial requirements. Cut - outs can also be introduced, where ever possible, to alter the resonance frequency.

Very little theoretical investigation for the determination of the natural frequencies has so far been made which is probably due to the complexity of the problem. Few experimental studies have been conducted but no systematic experimental investigation has been made to determine the effect of cut - out sizes, shapes, cut - out eccentricity etc.

1.2 Previous Work

Most of the theoretical and experimental work done on vibrations of rectangular plates with cut - outs is limited to only square plates with circular or square cut - outs.

Kumai[1] found three natural frequencies of vibrations of square plates with circular cut - outs analytically as well as experimentally for the simply supported and clamped boundary conditions. He computed natural frequencies for a given mode shape by means of superpositicn of solutions of circular ring plates satisfying the boundary conditions ·t several points alrng outside edges of the square plate. He allowed small residual deflections at some portions of the boundary of the square plate. He also conducted experiments on thin celluloid plates.

Takahashi[2] analysed the problem of rectangular plates with circular cut - outs and all ends clamped using Rayleigh - Ritz method. He used deflection function which are the product of the beam functions. He gave frequency parameter for several aspect ratios and cut - out sizes.

Joga Rao and Pickett[3] used energy method to find out the fundamental frequency of vibrations of square plates with circular and square cut - outs. They modified the deflection functions of plates without cut - outs to include one term giving appropriate singularity for the cut - out. They have given results for simply supported square plates with circular cut - outs and free rectangular plates with circular and square cut - outs.

Paramshivam[4] used grid frame work model to find out natural frequencies for clamped and simply supported square plates with square cut - outs. Fundamental

frequencies for various cut - out sizes have been determined. They also found higher frequencies of vibrations of a plate with cut - out size half that of the plate.

Anderson et. al.[5] computed natural frequencies of vibrations of clamped square plates with circular cut - outs using finite element method. They used triangular elements. The elements were taken to be finer near the cut - out boundary.

Kristiansen and Werner[6] have found fundamental frequencies of clamped square plates with various types of cut - outs using Rayleigh method. Product of beam functions neglecting the presence of cut - out has been used as deflection functions.

1.3 Present Work

From the literature survey it is clear that very little work has been done on the vibrations of rectangular plates with rectangular cut - outs. Since no systematic experimental work has been done to determine the natural frequencies for a plate with cut - outs, an experimental study has been carried out.

An experimental set-up was designed and fabricated for clamping plates with various aspect ratios. The plates were excited acoustically. The investigatin has been carried out to study the effect of various parameters viz. cut-out size, eccentricity, cut-out shape etc. The experimental values are

[redacted]

plotted in terms of non - dimensional frequency parameter against the cut-out size parameter. Results are compared with available theoretical and experimental results.

Chapter II contains the description of the experimental set-up. In Chapter III experimental results have been discussed. In Chapter IV concluding remarks and scope for further work have been discussed.

[redacted]

CHAPTER II

EXPERIMENTAL SET - UP

Experimental work has been carried out to study effect of cut outs on the vibrations of plates. Experimental set - up and procedures have been described in this chapter.

2.1 Excitation of Plates

To obtain the frequency at which the resonance occurs, plates can be excited

(a) acoustically

(b) by an electrodynamic shaker

(c) by variable frequency air - jet.

In the present work the plates were excited acoustically by means of a loud speaker. This was done due to the following reasons :

(1) In case of excitation by an electrodynamic shaker, the whole fixture with plate has to be mounted on the shaker which may not be always possible because of the force limitations of the shaker.

(2) A rod mounted on the shaker and attached to the plate can also be used for excitation. But the natural frequency of the plates may be affected by the contact of the rod with the plate.

[redacted]

(3) In case of variable frequency air jet excitation the mass of air striking the plate will affect the frequency of plates. The arrangement is quite complex as continuous air supply is always required.

An Oscillator was used to produce sinusoidal signal. Frequency and aplitude of the signal can be varied. The signal was amplified using an audio - power amplifier. Provision was made to keep the speaker above or below the plate.

2.2 Measurement of Natural Frequency

A contactless vibration pick-up (type mv - 2000) was used to find the frequencies of the vibrating plate. This was preferred over a strain gage because the latter cannot be used to find the mode shapes and the higher frequencies. The pick up generates an electrical output which is of the order of milli volts. The signal from the pick-up was amplified and fed to a timer and it was also displayed on a cathode ray oscilloscope. The amplification was necessary because of the trigger voltage limitations of the timer.

The frequency was found from the timer when the pick-up output is maximum, the out-put being displayed on a CRO.

2.3 Determination of Mode Shapes

Mode shapes were plotted by the use of a pantograph. At one of its ends the contactless pick up was mounted and at the other end a lead point was mounted for tracing purposes.

[redacted]

To plot the mode shapes, the frequency of excitation was changed slowly in the oscillator. As the exciting frequency approaches the natural frequency, the output signal from the pick-up increases enormously which was seen on the oscilloscope. The frequency was adjusted until the out-put signal was maximum. The timer directly gives the natural frequency.

This frequency was then kept constant and the pick-up moved above the surface keeping a constant distance from the plate. As the pick-up moves towards a nodal line, the output from the pick-up goes on decreasing and as it crosses the nodal line, the output again increases and becomes out of phase of the previous signal (i.e. when the pick-up did not cross the nodal line). The point for minimum output was marked on a paper by the lead point at the other end of the pantograph. The pick up was again moved over the surface for plotting similar points. These points were joined by a continuous curve to give the nodal line. However, the plot thus obtained was antisymmetric to the nodal pattern of the plate.

Mode shapes could also be seen by sprinkling fine sand particles over the plate.

The sound level of the speaker in operation and of the background noise were measured by a sound - level meter.

2.4 Clamping Arrangements

A fixture was designed for mounting the clamped plates. Details of the fixture are shown in Figure 1. It consists of four vertical members bolted to the foundation and four horizontal members supported by the vertical members. Two of the horizontal members are fixed and the two are movable to adjust for the different sizes of plates.

The plates were clamped by means of two sets of thick mild steel flats. The plates were kept between these two flats and tightened using bolts and nuts. Equal tightening was achieved by the use of a torque wrench. Clamping arrangement is shown in Figure 2.

2.5 Test Plates

Test plates were cut from Aluminium sheets of thickness 0.0364". Table 1 gives different types of plates used. A typical test plate is shown in Figure 3.

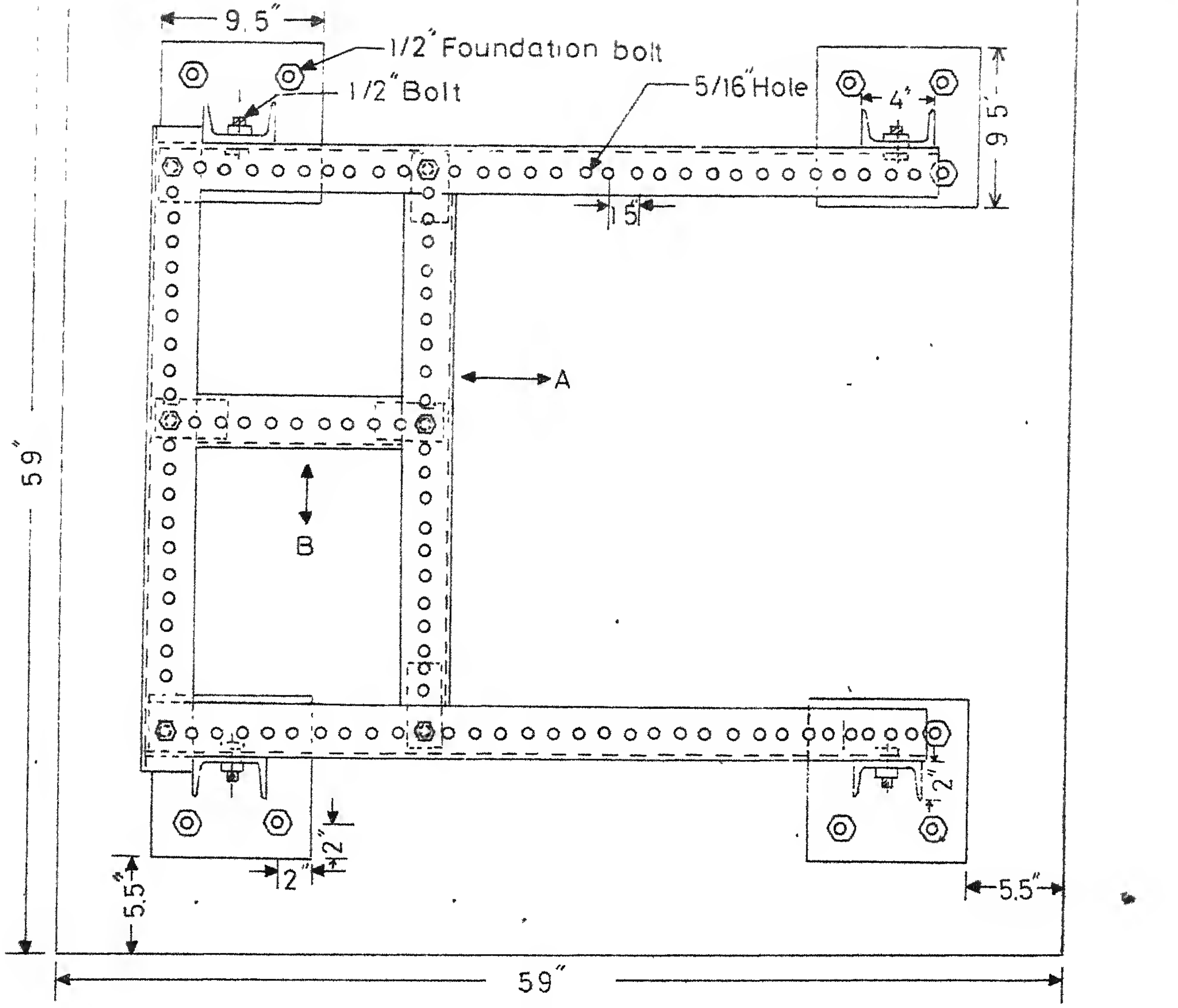
2.6 Specifications of The Instruments

(a) Low - distortion Oscillator :

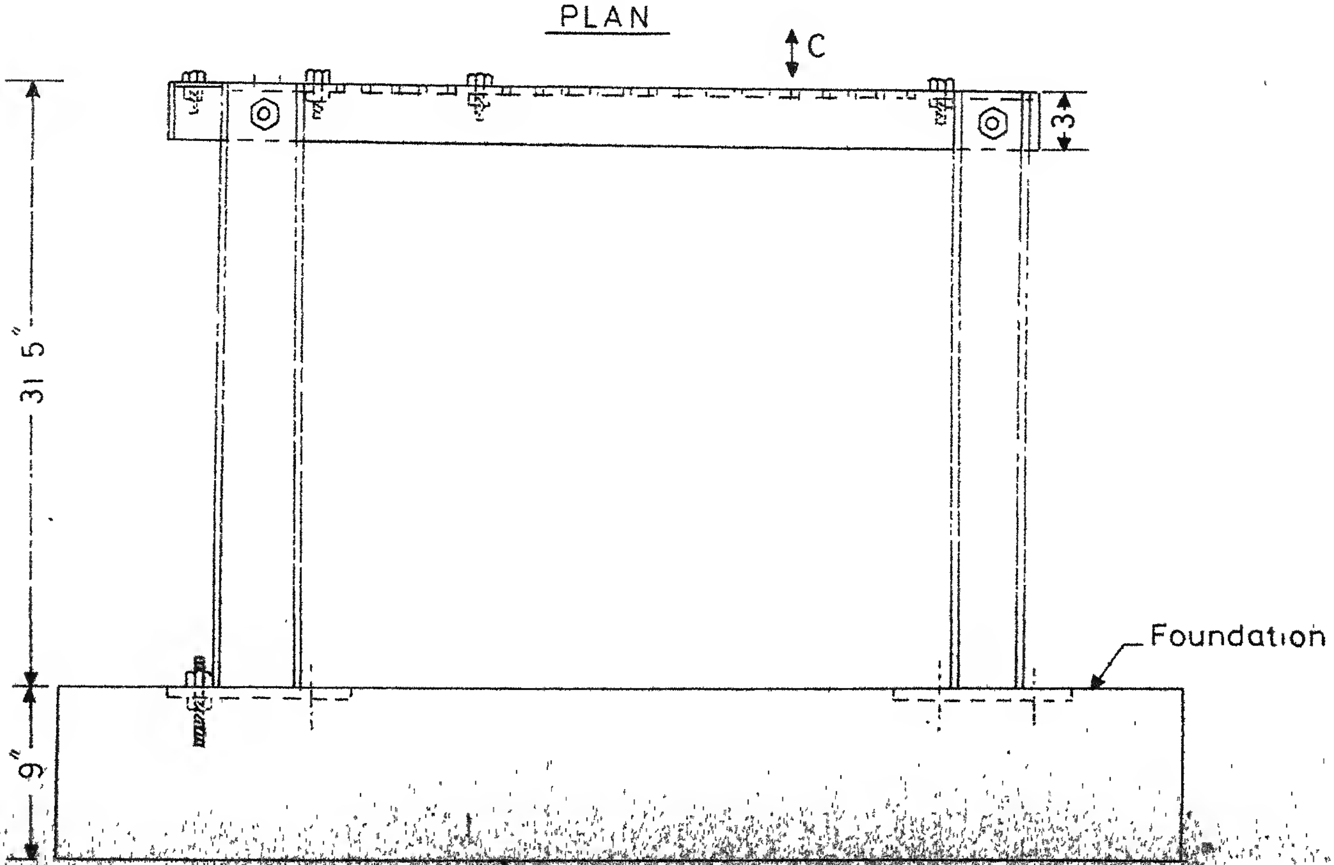
Frequency : 1.5 to 150 k c/s continuous coverage in 5 ranges at decade interval.

Accuracy : ± 3 per cent

Distortion: Harmonic content of ine wave is less than 0.15% at 1 kc/sec.



PLAN



ELEVATION

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OF FIXTURE

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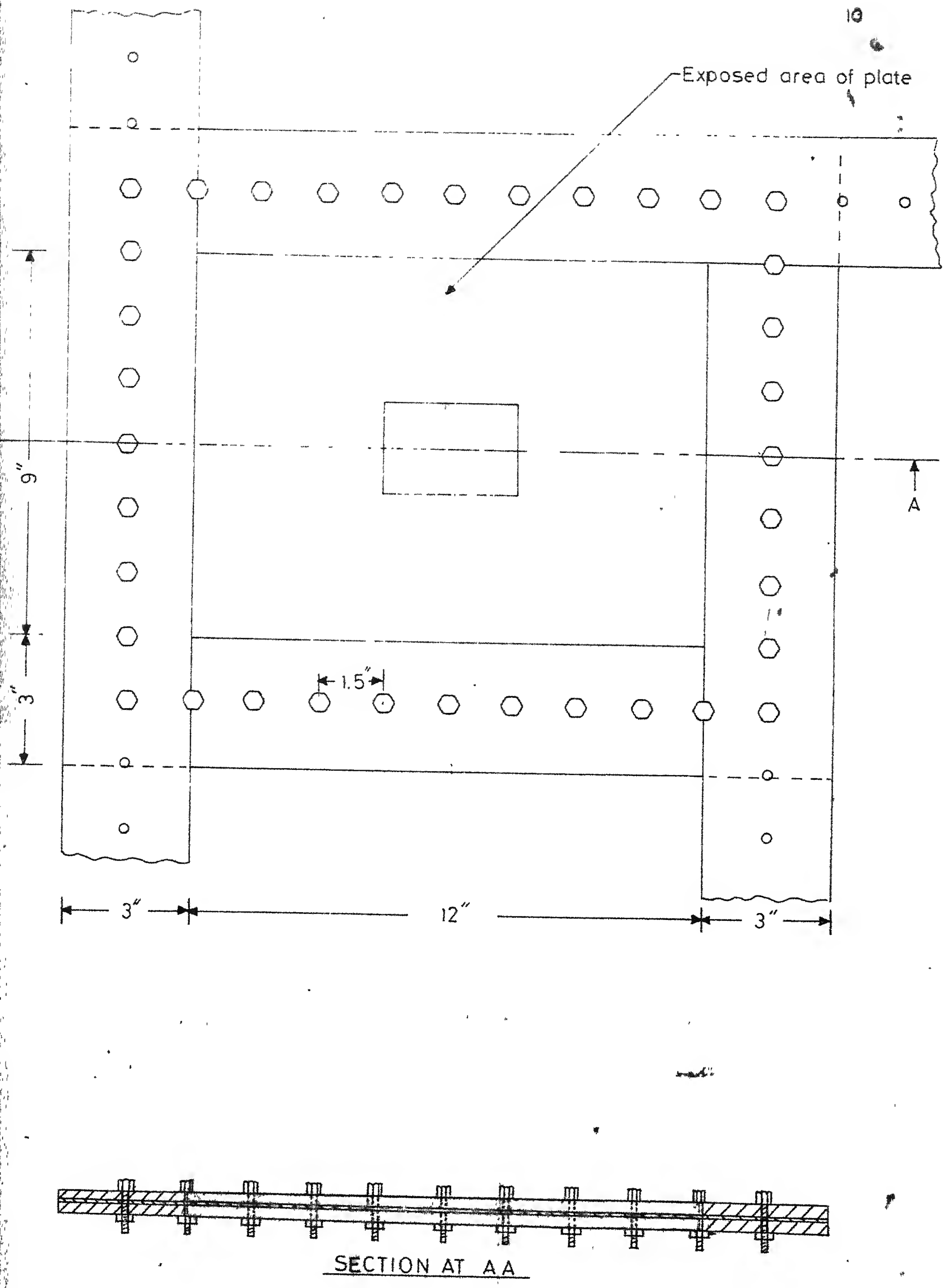

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FIG. 2. DETAILS OF CLAMPING.

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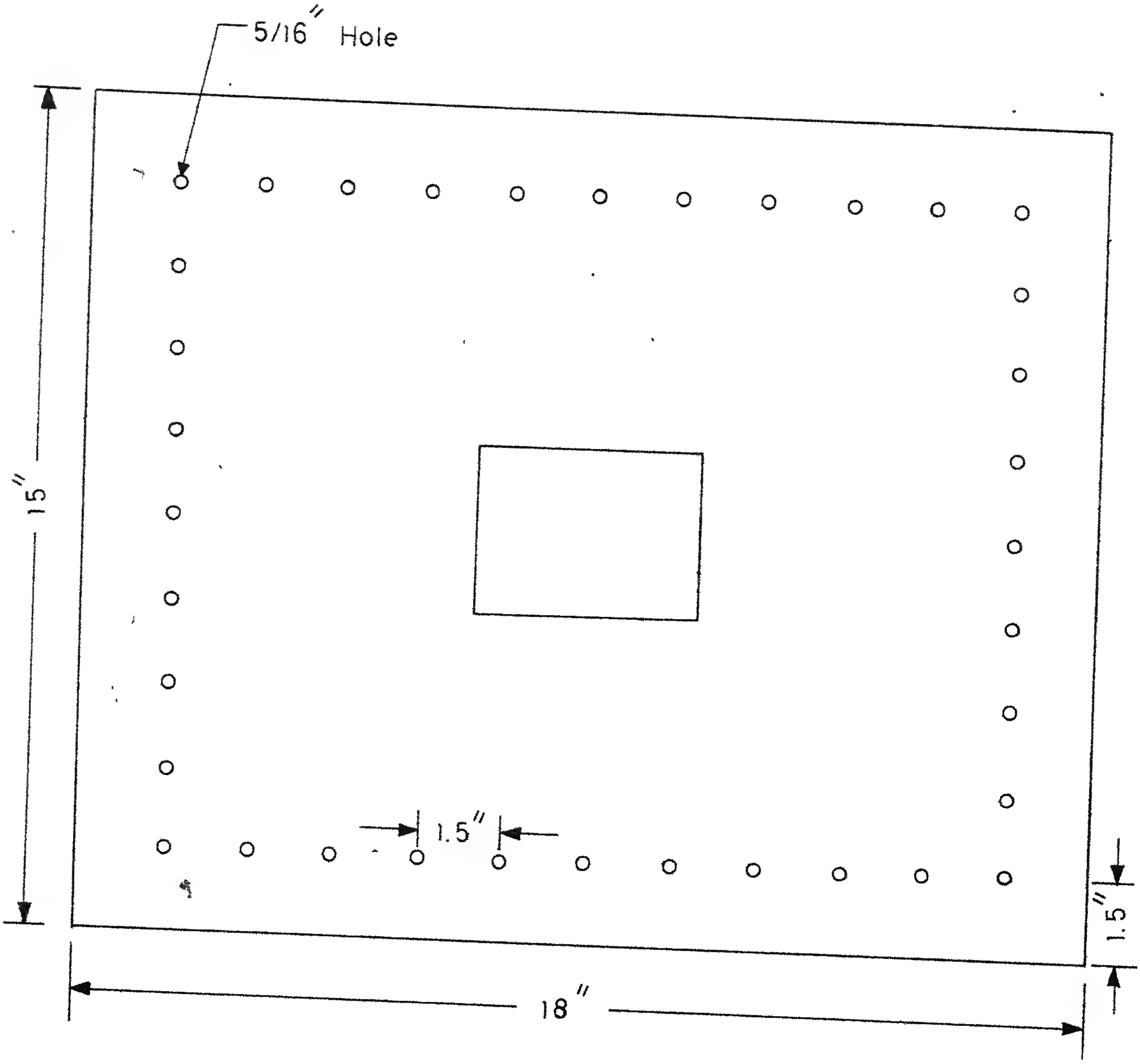

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FIG. 3. TEST PLATE

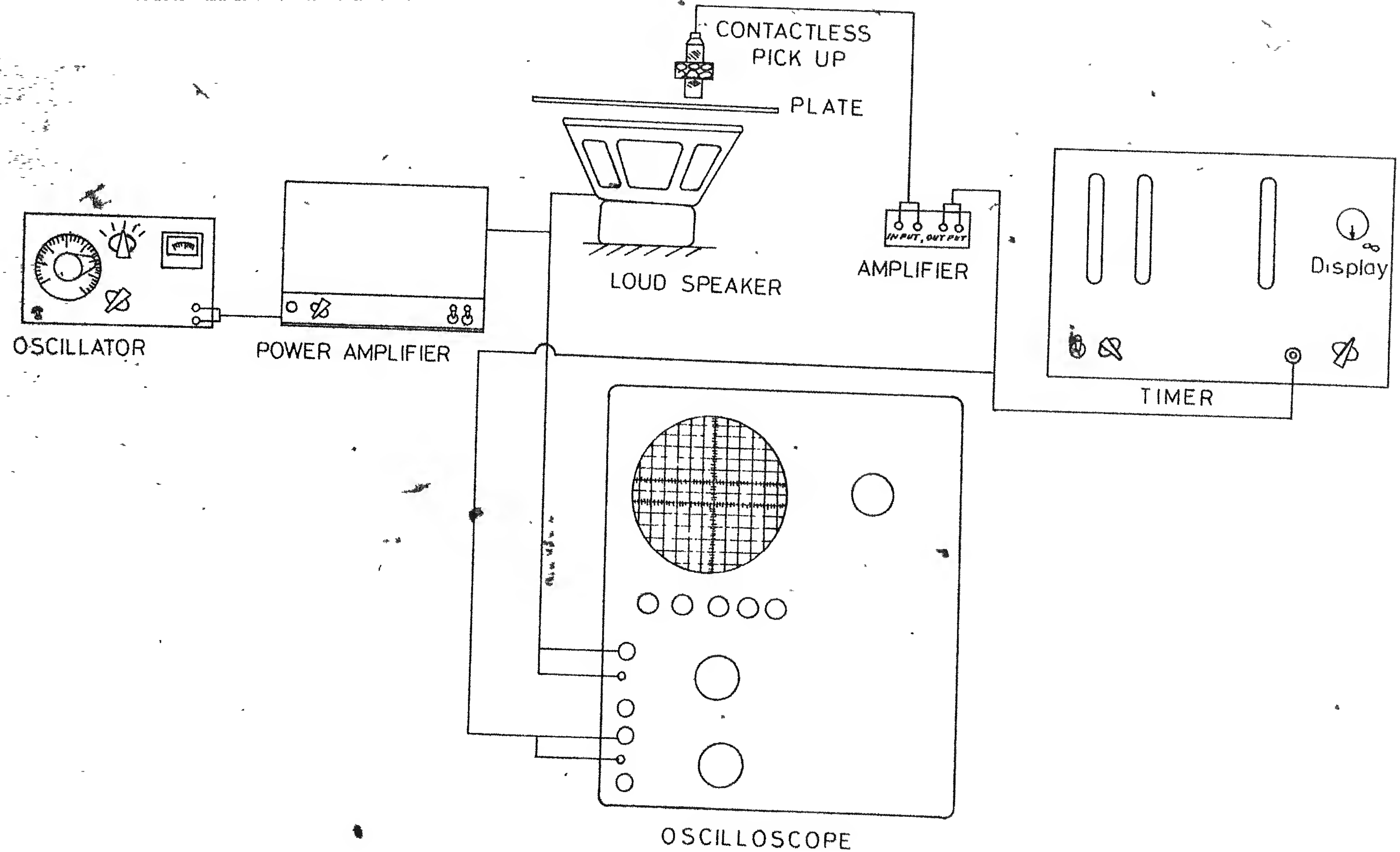

FIG: 4. LINE SKETCH OF EXPERIMENTAL SET-UP.

[redacted]

Output impedance : (a) 600 $\Omega \pm 2\%$ via attenuator

(b) less than 250 Ω direct.

SYSTRONICS, AHEMEDABAD (INDIA)

(b) Power Amplifier (CH 75) :

Maximum output : 80 watts

CHICAGO RADIO (U.S.A)

(c) Loud - Speaker (Cone - type) :

Diameter : 16 inches

Power output : 16 watts

Resistance of the coil : 6.5 Ω

JENSON (U.S.A.)

(d) Vibration Pick-up (Type MV 2000) :

Dynamic Frequency Range	:	2 c/s to 1000 c/s
Optimal Gap	:	2 mm
Vibration Amplitude	:	$\pm$ 1.5 mm maximum
Mounting	:	Two hexagonal nuts, 19 mm are provided with pick - up to fix with a clamp.
Dimensions	:	Cylindrical length 45 mm diameter 19 mm
Weight	:	130 gms

NAL BANGALORE (INDIA)

(e) * Dual Beam Oscilloscope (Type 502 A)

The oscilloscope provides linear dual beam display with a wide range of sweep rates combined with sweep magnifier and high input sensitivity. The triggering can be done by the input signal to obtain stable display.

In the vertical deflection system there are seventeen calibrated deflection factors from 0.1 mv/cm to 20 v/cm accurate within 3 percent.

In horizontal deflection system there are twentyone calibrated sweep rates from 1 μsec/cm to 5 sec/cm.

TEKTRONIX INC BEAVENTON OREGON (U.S.A.)

(f) * Universal Eput and Timer

Timer is an electronic counter which can perform a wide variety of frequency and time measurements. The instrument counts number of events which occur during a precise time interval. Since each cycle of input signal represents one event, this measures the frequency of input signal.

Frequency Range	-	10 cps to 11.5 Mcs
Counting period	-	Selectable in decade steps from 1 μ sec to 10 secs. with a manual extension beyond 10 seconds.
Accuracy	-	$\pm$ one count.

BECKMAN INSTRUMENTS (U.S.A)

[redacted]

(g) Sound Level Meter (Type 1565-A)

Sound Level Range	-	40 db to 140 db
Power supply	-	$1\frac{1}{2}$ volt (flash light cell)
Operating temperature range	-	10 °C to 50 °C
Storage temperature range	-	30 °C to 40 °C
Operating humidity range	-	0 to 90% Rh

GENERAL RADIO CO. (U.S.A.)

[redacted]

CHAPTER III

RESULTS AND DISCUSSIONS

In this chapter the theoretical results as given by previous investigators are compared with experimental results. A discussion of various parameters is also made.

3.1 Experimental Results

For plates with different cut-out sizes and shapes (as given in Table 1), natural frequencies and mode shapes were recorded and plotted respectively. Natural frequencies have been given in Tables 2 to 10 and mode shapes in Figures 5 to 13.

3.2 Graphical Plots

In figures 14 to 18, non - dimensional frequency parameter c (defined as the ratio of the natural frequency of a plate to the fundamental frequency of the plate without cut-out) has been plotted against non-dimensional cut-out size parameter (defined as the ratio of the length of cut-out to the length of the plate) for aspect ratios 1.00, 0.89, 0.75, 0.625, 0.5.

The non-dimensional parameter k of fundamental frequency for theoretical and experimental cases is plotted against the parameter r in Figure 19.

[redacted]

In Figures 20 to 24, non dimensional frequency parameter k for first three natural frequencies has been plotted against cut-out size parameter r for various aspect ratios.

3.3 Effect of Clamping

In order to find the effect of clamping, natural frequencies of plates (aspect ratios 0.5) were recorded for various values of torques applied to bolts. These are given in Table 11. It is seen from the table that as the torque applied to bolts is increased, frequencies also increase. But there is no appreciable change in frequencies as torque is increased from 10 ft. lb. to 15 ft. lb. (the maximum change being 2 cycles per second only).

It is not possible to get perfect "clamping" mechanically. Laura et. al[7] have suggested the use of effective length to take into account some of the imperfections. They defined effective length as given below.

Effective length = $a + s/n$

where s = clamped length

n = a constant for a particular clamping.

In Table 12 experimental results are compared with theoretical results for a cantilever plate using actual length. Maximum difference in frequency was found to be 15.3 percent. For various values of n ranging from 2 to 10, computed frequencies were compared with experimental values. It was found that theoretical values compare well with experimental values for $n = 7$

(Table 13a). Similarly, taking n = 7 theoretical results for clamped - clamped plate agreed well with those experimentally found (Table 13b).

3.4 Effect of Back-ground Noise

Noise levels were measured with a sound level meter, when the loud speaker was operating and when put - off. The measured sound levels were 105 db and 50 db respectively. It is seen that the sound level when speaker is operating was much higher than the back ground noise. Moreover, the back-ground noise was found to have no effect on the response of the plate as no output was obtained from the pick-up with loud speaker put off. This shows that the background noise did not affect the determination of frequency during the experiment.

3.5 Comparison of Results

Experimental results of plates without cut-outs are compared with theoretical results (taking n = 7) and it is seen from Table 13b. that these results agree well.

Theoretical frequency parameter k of fundamental frequency of square plates with square cut-outs and higher frequencies (r = .5) given by Paramsivam[4] are compared with present experimental results (Fig. 19 and Table 14 (a), (b)). Paramsivam used grid frame work analogy to calculate the frequencies. It is seen that the theoretical results are lower than the experimental values.

This may be because of the fact that a 8 x 8 mesh mesh used is not sufficient in the determination of the plate natural frequencies. Theoretical value of the natural frequency is shown to increase with the mesh size and this value approaches the present experimental results (Table 14c).

Takahashi[2] has given the theoretical and experimental values of fundamental frequency of rectangular plates (aspect ratio = 0.5) with circular cut-outs for the value of r upto 0.2. Experimentally obtained value of k for same area ratio compare well with his theoretical and experimental results (Table 14d).

3.6 Effect of Cut-outs on Frequency

3.6.1 Effect of Cut-out Size

For plates with symmetrical cut-outs, it is observed that the fundamental frequency increases with the increase in cut-out size for all aspect ratios. For a clamped plate the mass is uniformly distributed but the stiffness is not uniform. In the center the stiffness is less as compared to the clamped edges. The reduction in stiffness is less compared to the reduction in mass due to removal of a portion of the plate and therefore, the frequency increases.

The variation of the frequency parameter k with respect to the size parameter r for rectangular plates with rectangular cut-outs is found to be similar to rectangular plates

[redacted]

with circular cut-outs as given theoretically by Takahashi[2].

In the case of second natural frequency, it is found that for small values of r (upto r = 0.1), the frequency parameters k and c (a frequency ratio parameter) increase. As r is increased further (from r = 0.1 to r = 0.35) these parameters decrease. Sharp decrease is seen for higher values of aspect ratio (1.0 to 0.90). But the decrease rate is lower for aspect ratio 0.625 to 0.5. Again, the parameters k and c increase for values of r more than 0.35. The results are compared with those obtained by Kumai.[1] In the results given by Kumai, unlike the present result, the frequency parameters do not increase for small values of r . For higher values of r , the trend is the same.

In the case of aspect ratios from 0.5 to 0.625, it is observed that the variation of k and c with respect to r for the third frequency is the same as that for fundamental frequency. For the aspect ratios 0.75 to 1.0, it is seen from the Figures (14, 15 and 16) that the behaviour of curves plotted for the frequency parameters (k and c) against the size parameter (r) is the same as for the curves obtained in the case of second natural frequency.

For plates with eccentric cut-outs, it is observed that the fundamental frequency decreases as cut-outs are shifted towards the clamped edges (Table 9). This is due to the

[redacted]

removal of the stiffer portion of the plate when cut-outs are shifted towards the clamped edges.

3.6.2 Effect of Cut-out Shape

The fundamental frequency is almost the same for different cut-out shapes (diamond, circular, rectangular, square) when the ratio of cut-out area to plate area is kept constant. Maximum variation in experimental results obtained is 5 percent.

Kristiansen and Werner[6] have found theoretically the fundamental frequency of square plates with various types of cut-outs (circular, square, diamond and elliptical). For the area ratios less than 0.64, circular cut-out gave the maximum value of frequency. However, the difference was negligible between circular and square cut-outs. For area ratio less than 0.2 the difference for circular, square and diamond cut-outs is negligible.

The present experimental values for the case of the area ratio 0.18 is compared with the above theoretical prediction and is found to be in agreement.

3.7 Effect of Cut-outs On Mode Shapes

It is observed that mode shapes for plates with different cut-out sizes are same as for plates without cut-outs with slight distortion as shown in Figures 8 to 13.

For plates with aspect ratio 0.625 and values of r between 0.266 to 0.534, one extra frequency is observed between

[redacted]

"second" and "third" modes. For this frequency, the mode shape was found to be similar to the "fourth" mode of a plate with no cut-out. This is due to the fact that the frequency for the third mode shape increases with increasing cut-out size (Figure 17) where as the frequency for the fourth mode shape decreases and therefore for $r = 0.266$ to $r = 0.534$, the frequency for fourth mode shape is found to lie between the frequencies for second and third mode shapes.

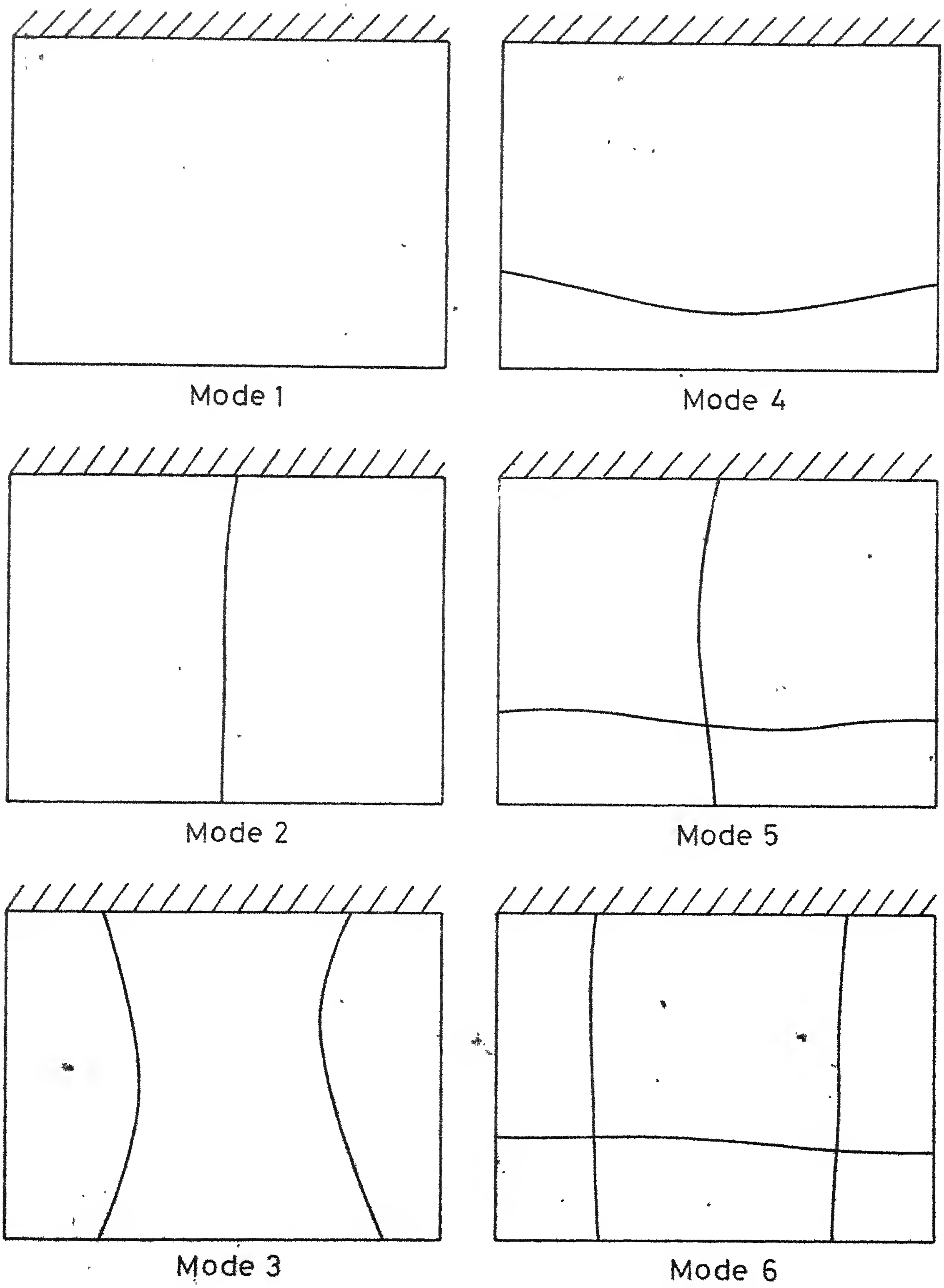

FIG.5. NODAL PATTERNS OF RECTANGULAR CANTILEVER PLATE. $a/b = 0.75$.

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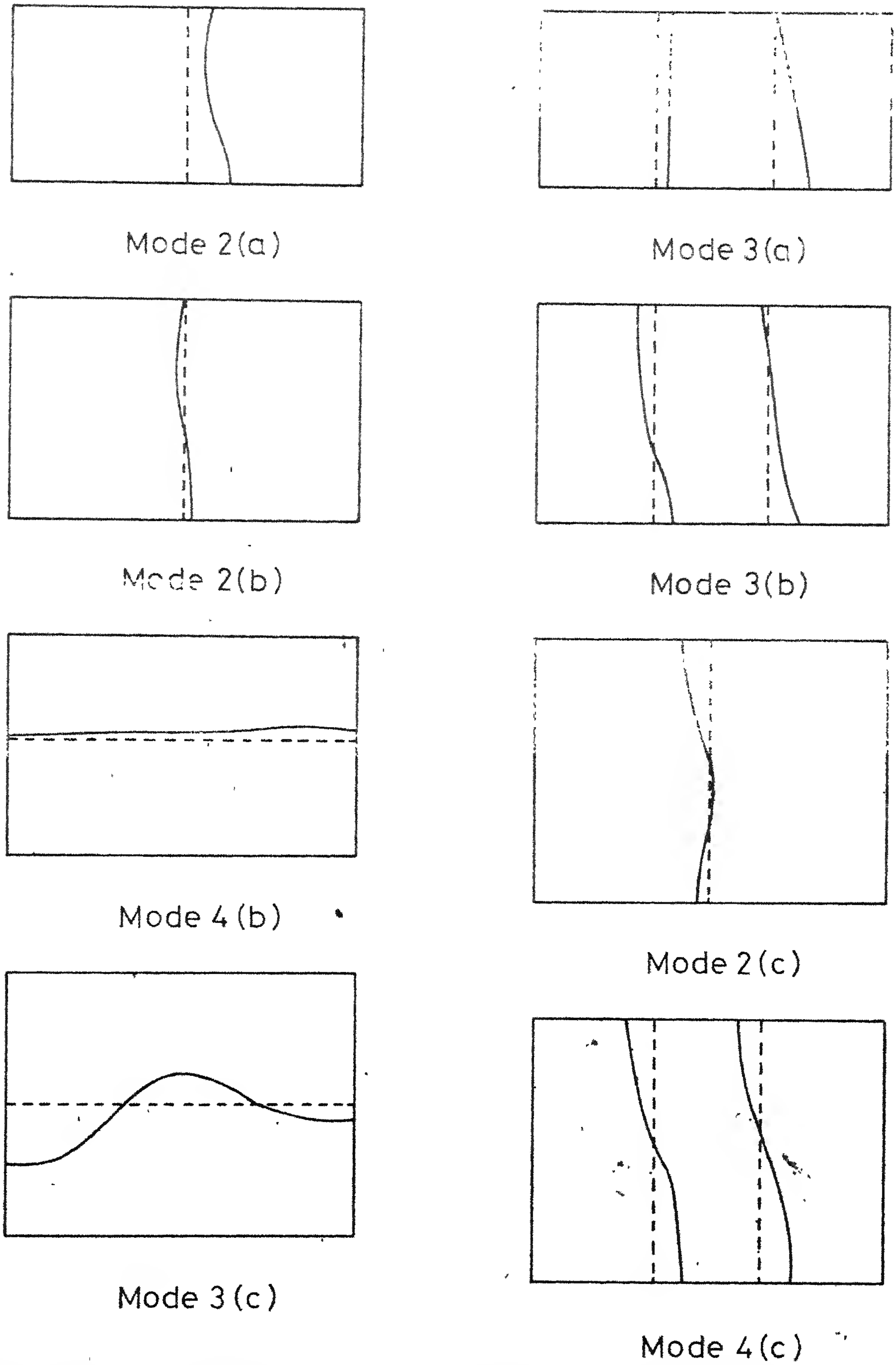


FIG.6. NODAL PATTERNS OF RECTANGULAR PLATES.

(a) a/b = 0.5 (b) a/b = 0.625 (c) a/b = 0.75

--- Theoretical Nodal Line. —— Experimental Nodal Line.

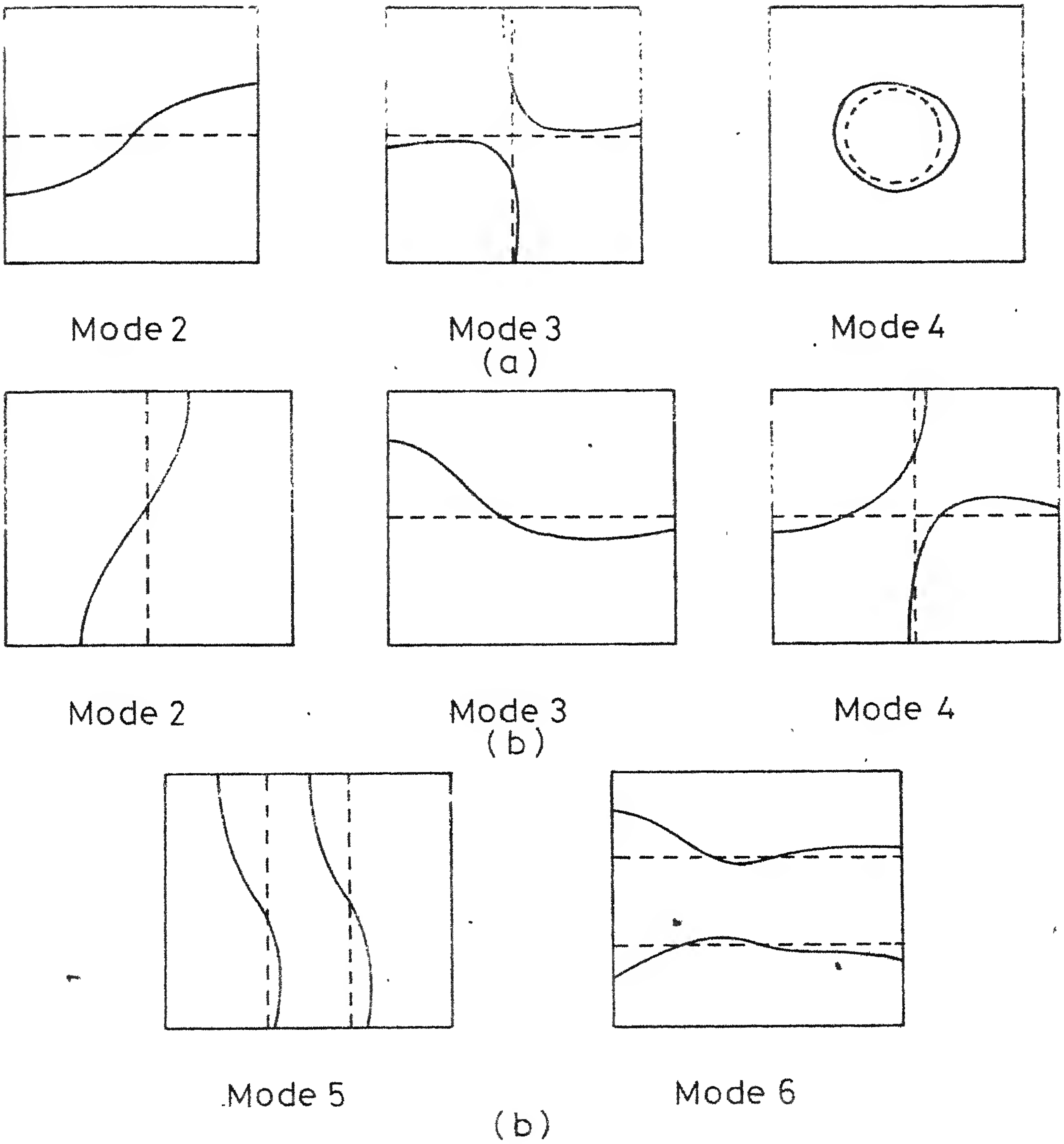


FIG. 7. NODAL PATTERNS OF RECTANGULAR PLATES.

(a) = a/b = 1. (b) a/b = 0.89.

[redacted]

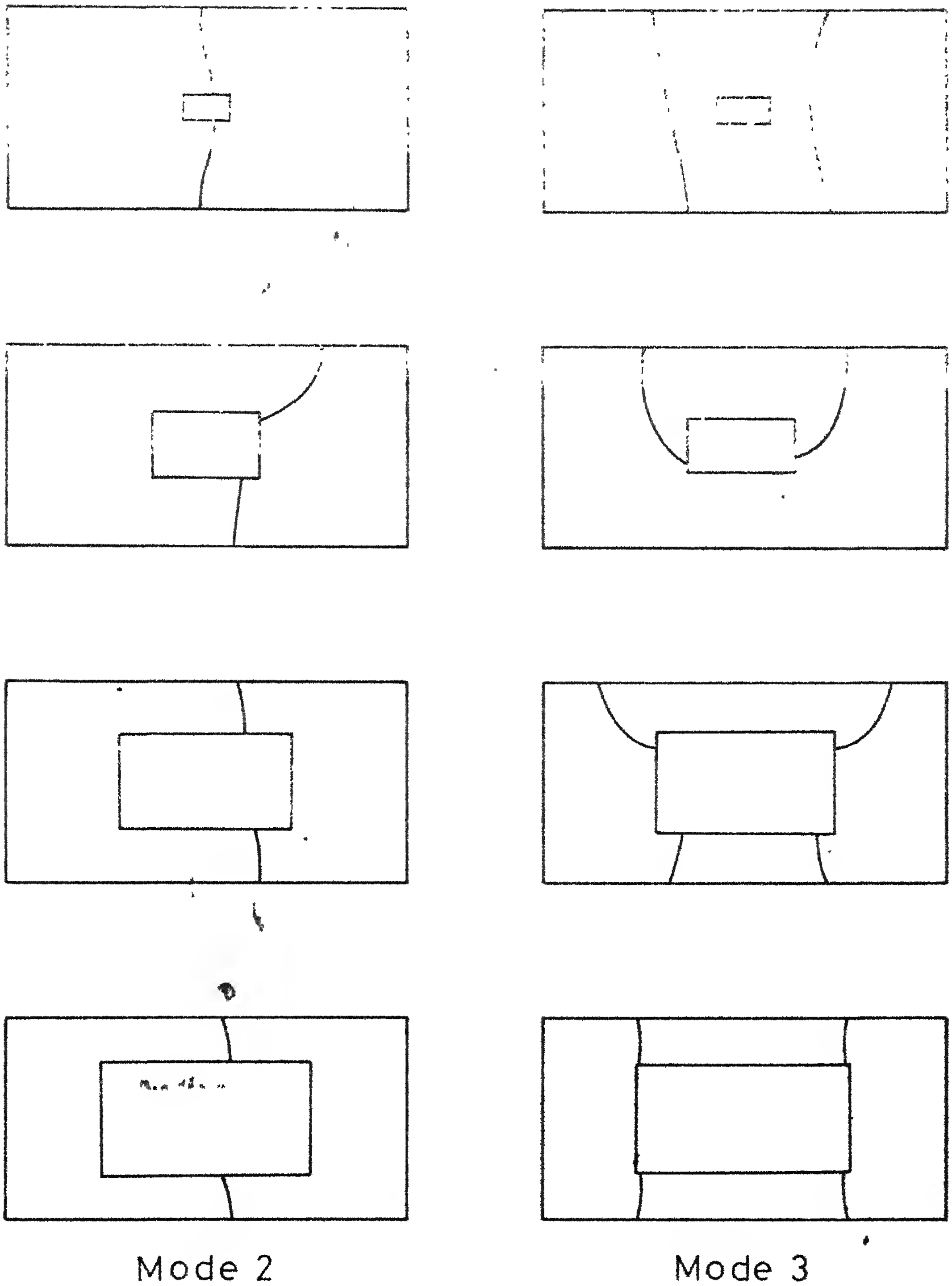


FIG. 8. NODAL PATTERNS OF RECTANGULAR PLATES WITH CUTOUTS. $a/b = 0.5$.

[redacted]

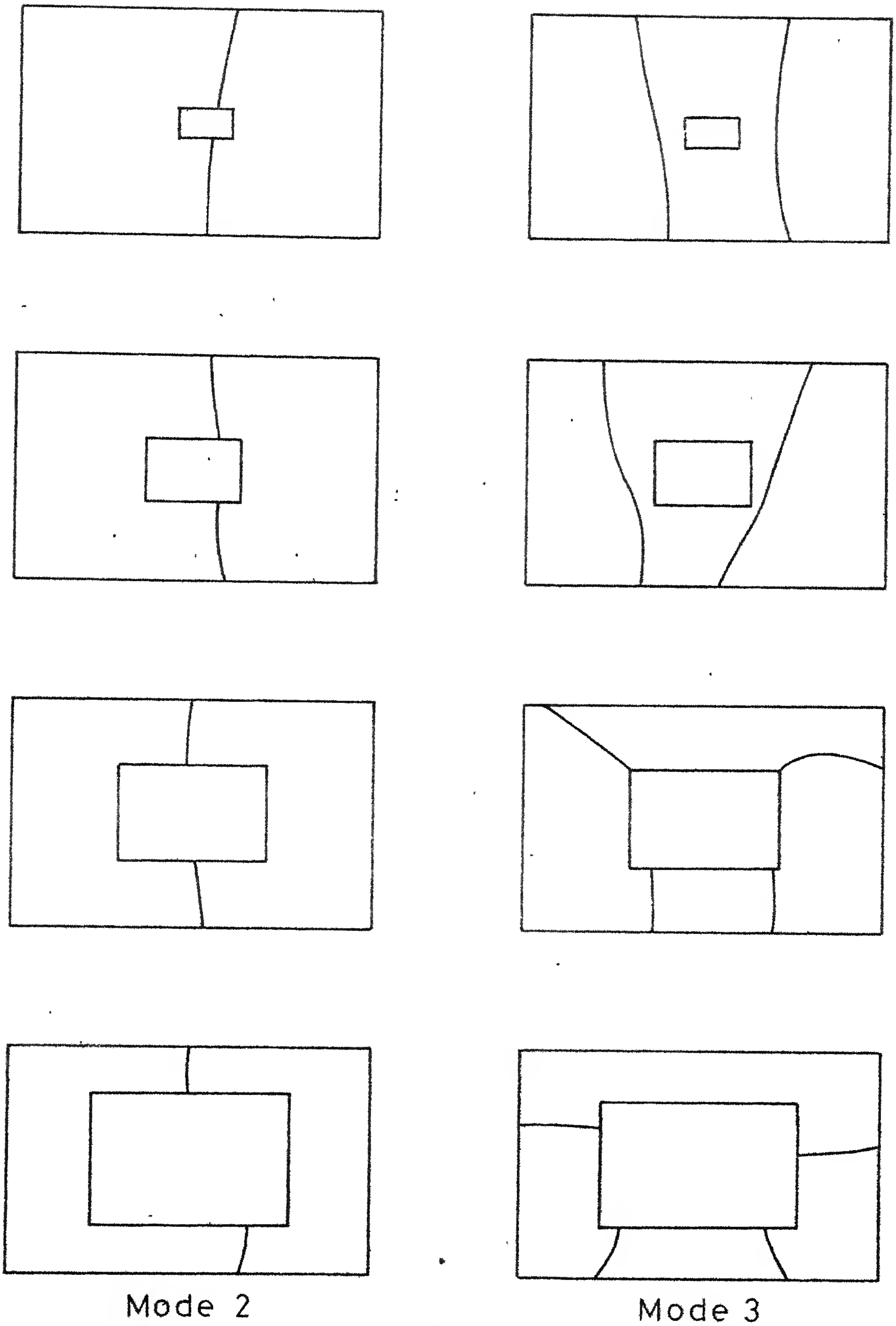
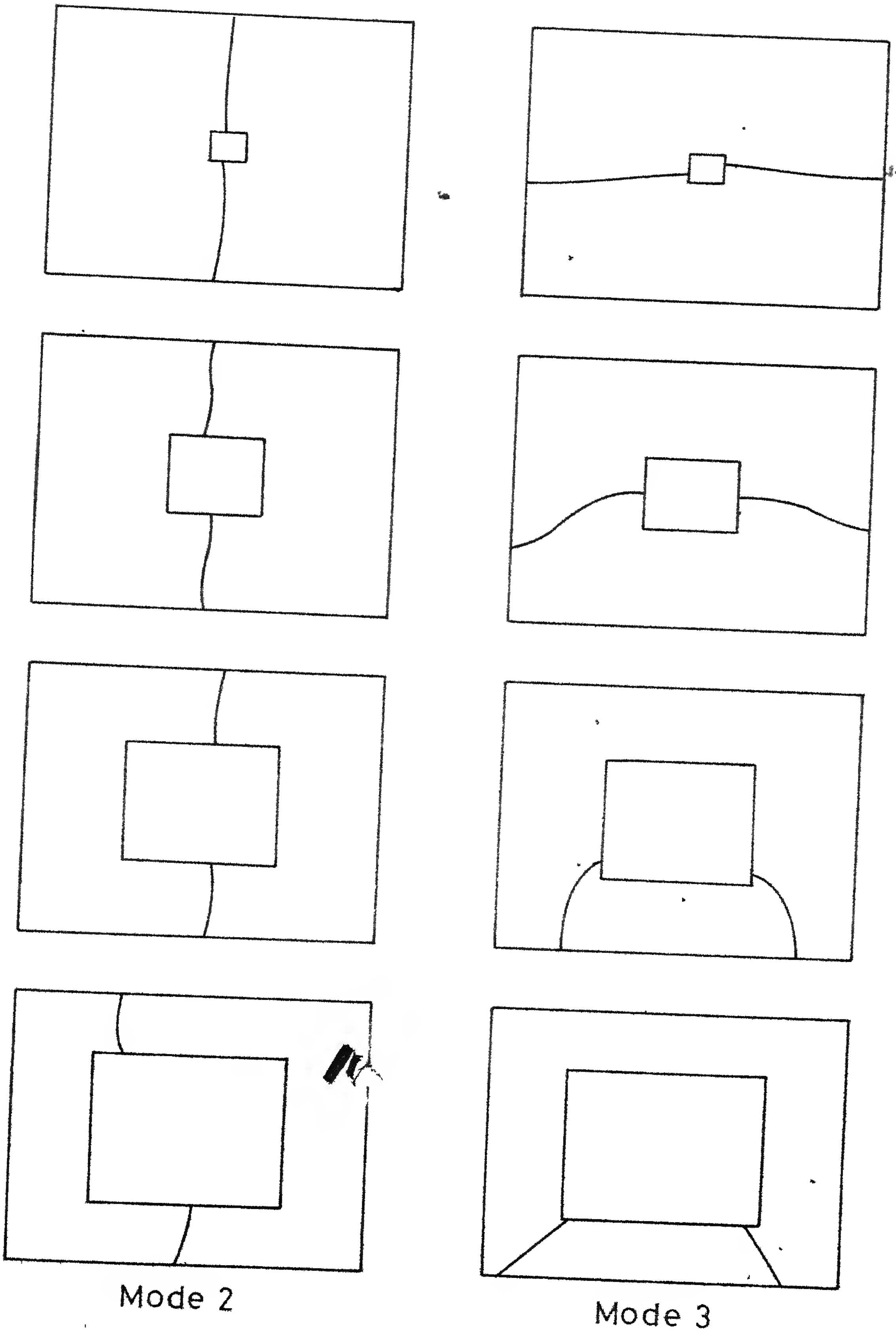


FIG.9. NODAL PATTERNS OF RECTANGULAR PLATES WITH CUTOUTS. $a/b = 0.625$.

[redacted]



). NODAL PATTERNS OF RECTANGULAR PLATES. a/b=0.75.

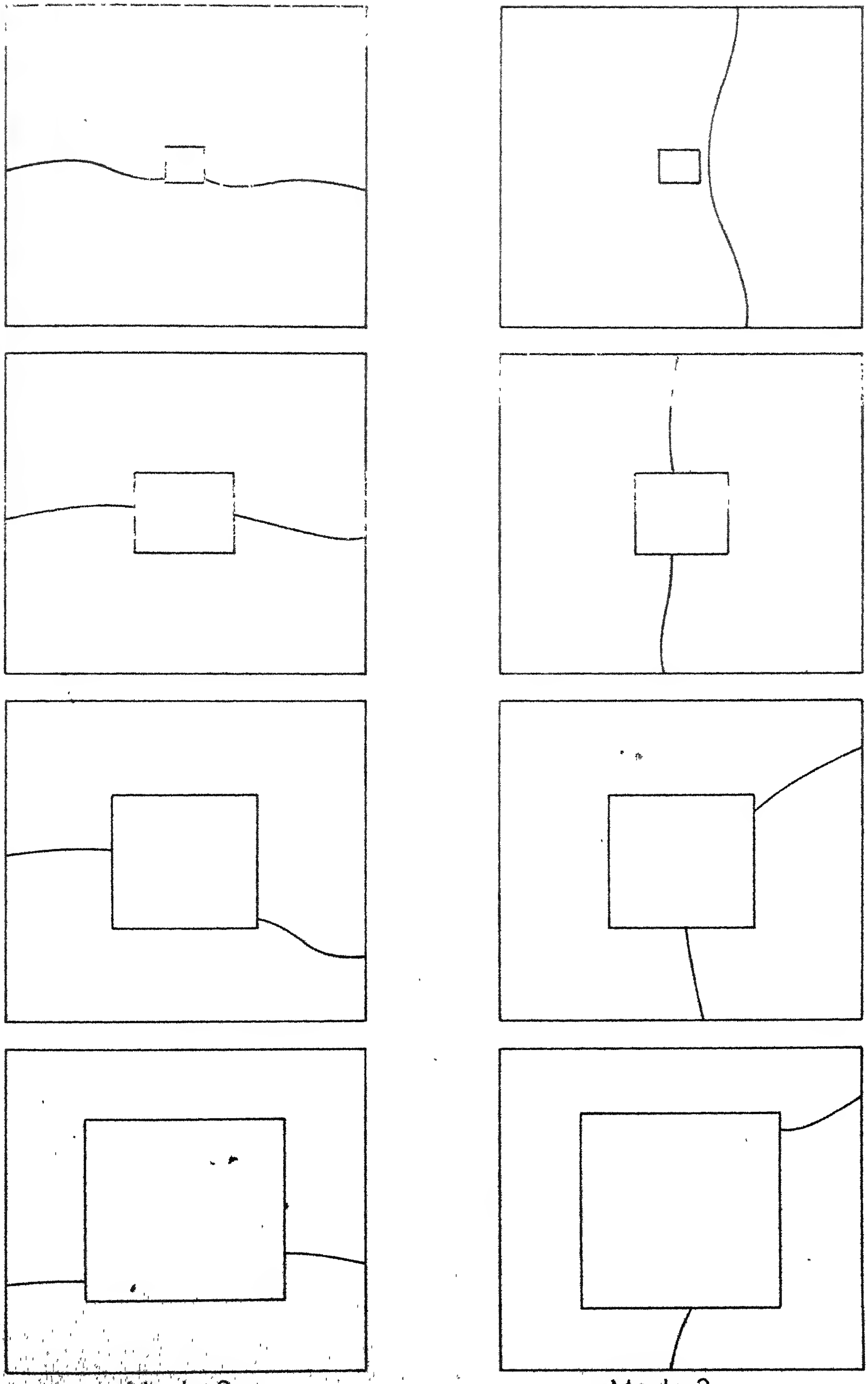
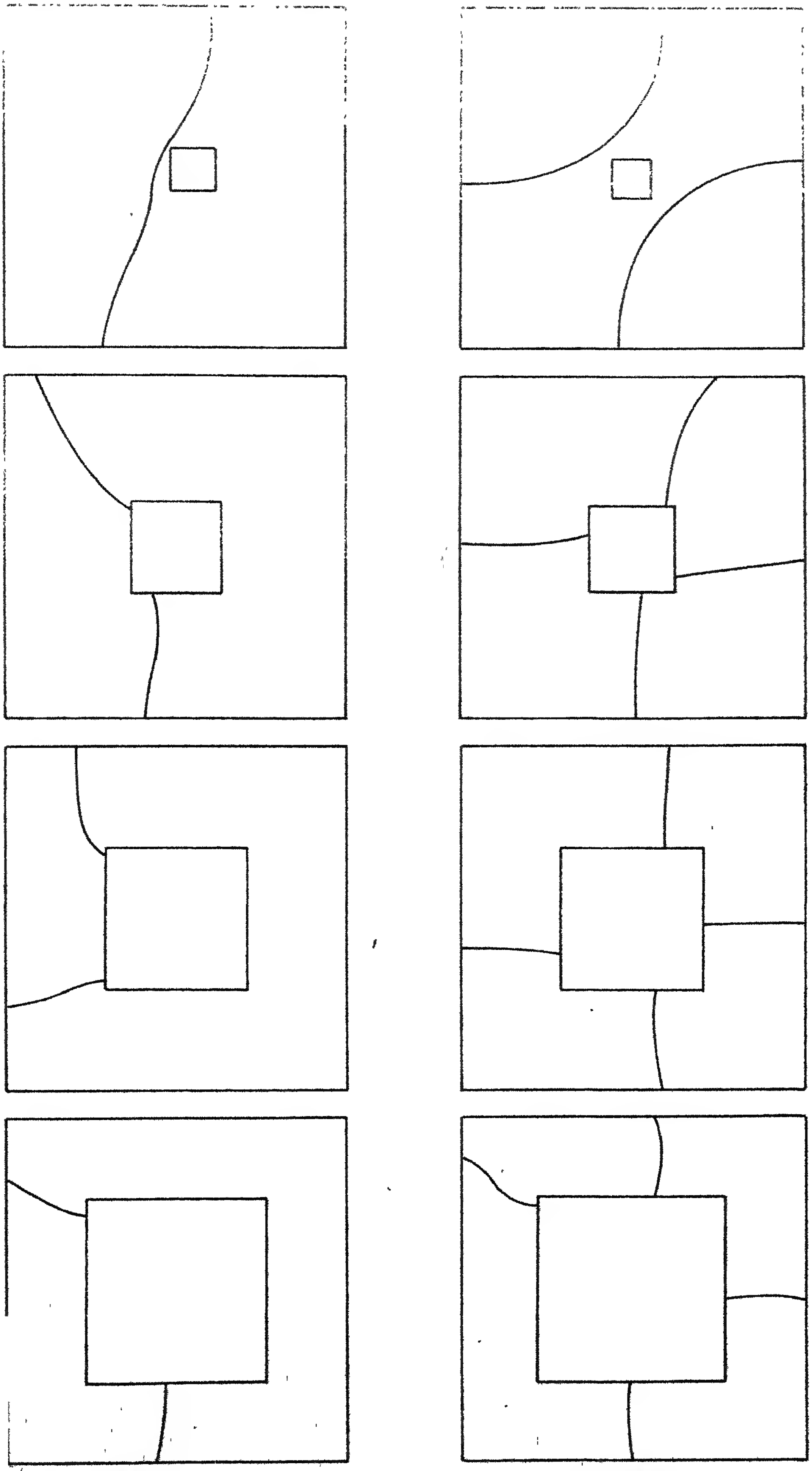
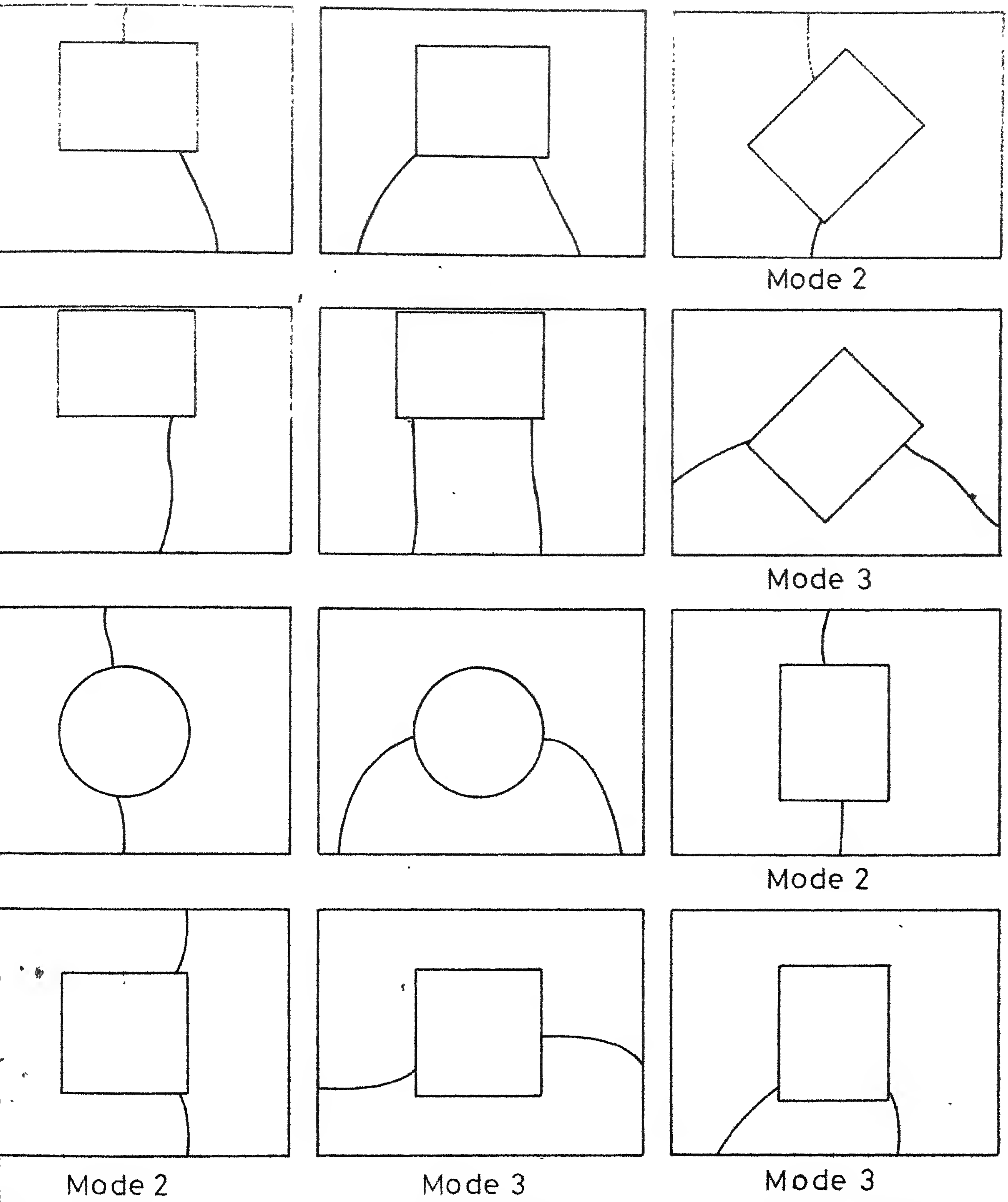


FIG.11 NODAL PATTERNS OF RECTANGULAR PLATES. $a/b=0.89$

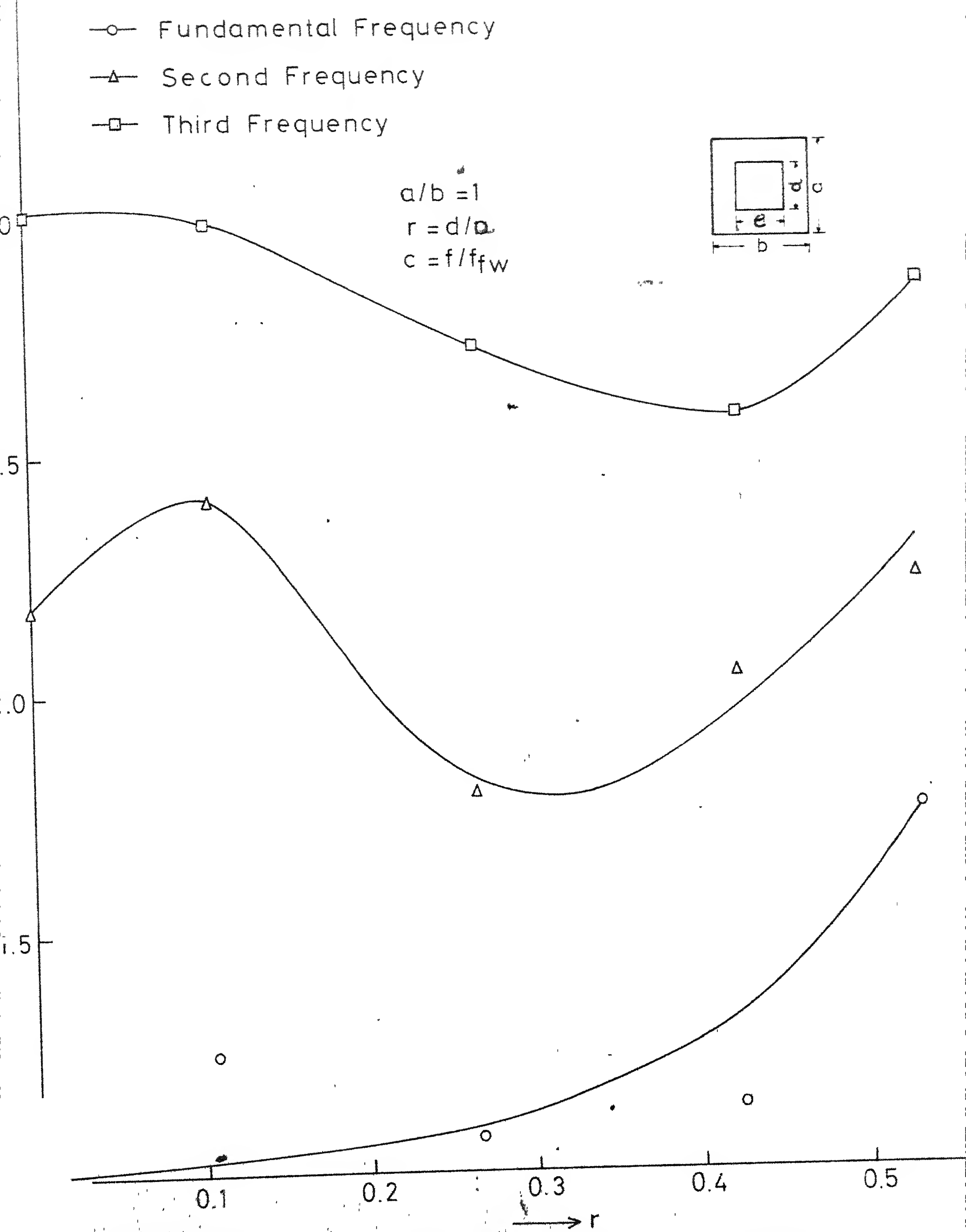
[redacted]



.12 NODAL PATTERNS OF RECTANGULAR PLATES. a/b=1



5.13. NODAL PATTERNS OF RECTANGULAR PLATES WITH VARIOUS TYPES OF CUTOUTS. $z = 0.18$.



14. FREQUENCY RATIO PARAMETER Vs SIZE PARAMETER

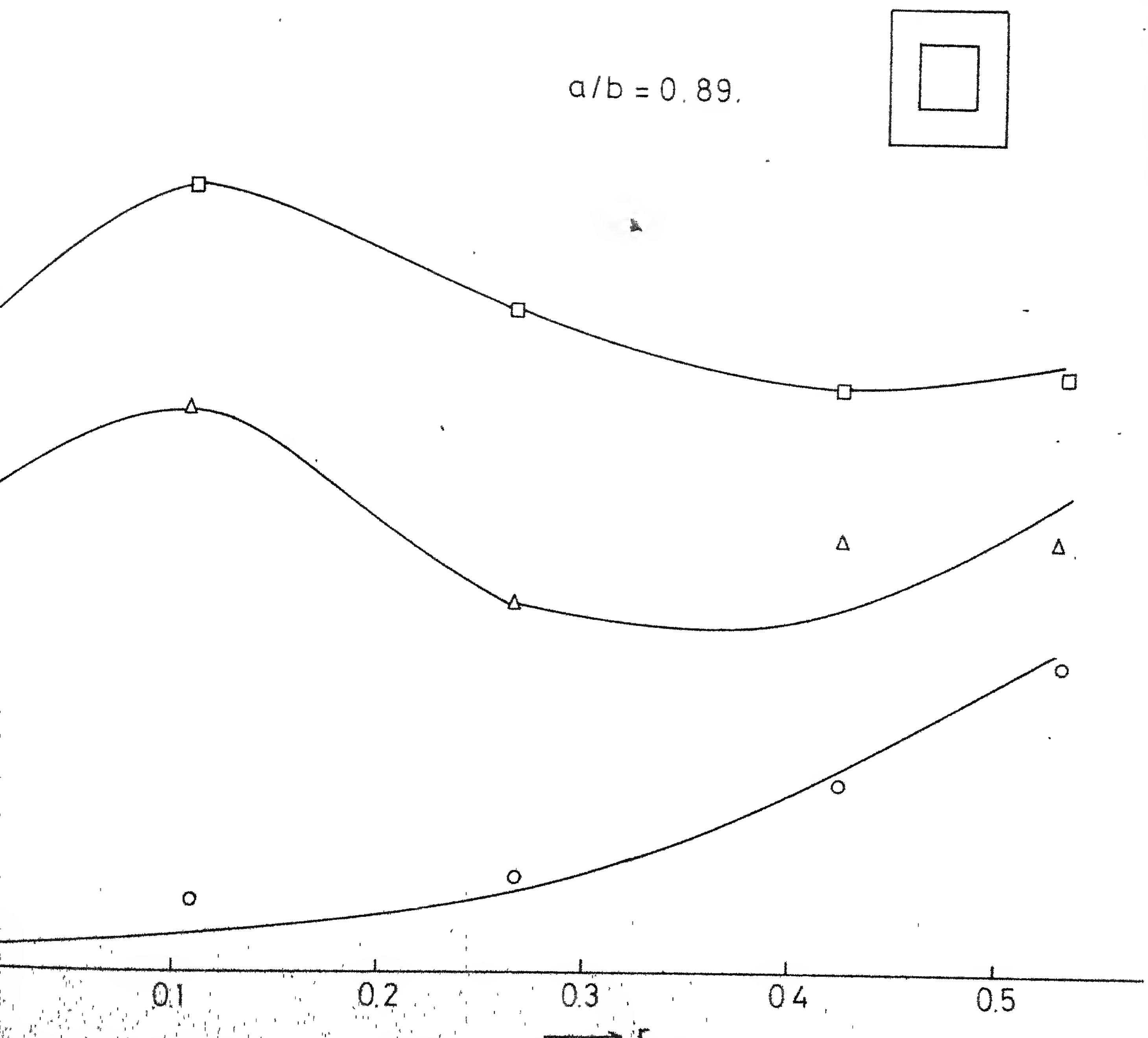


FIG.15 FREQUENCY RATIO PARAMETER Vs SIZE PARAMETER

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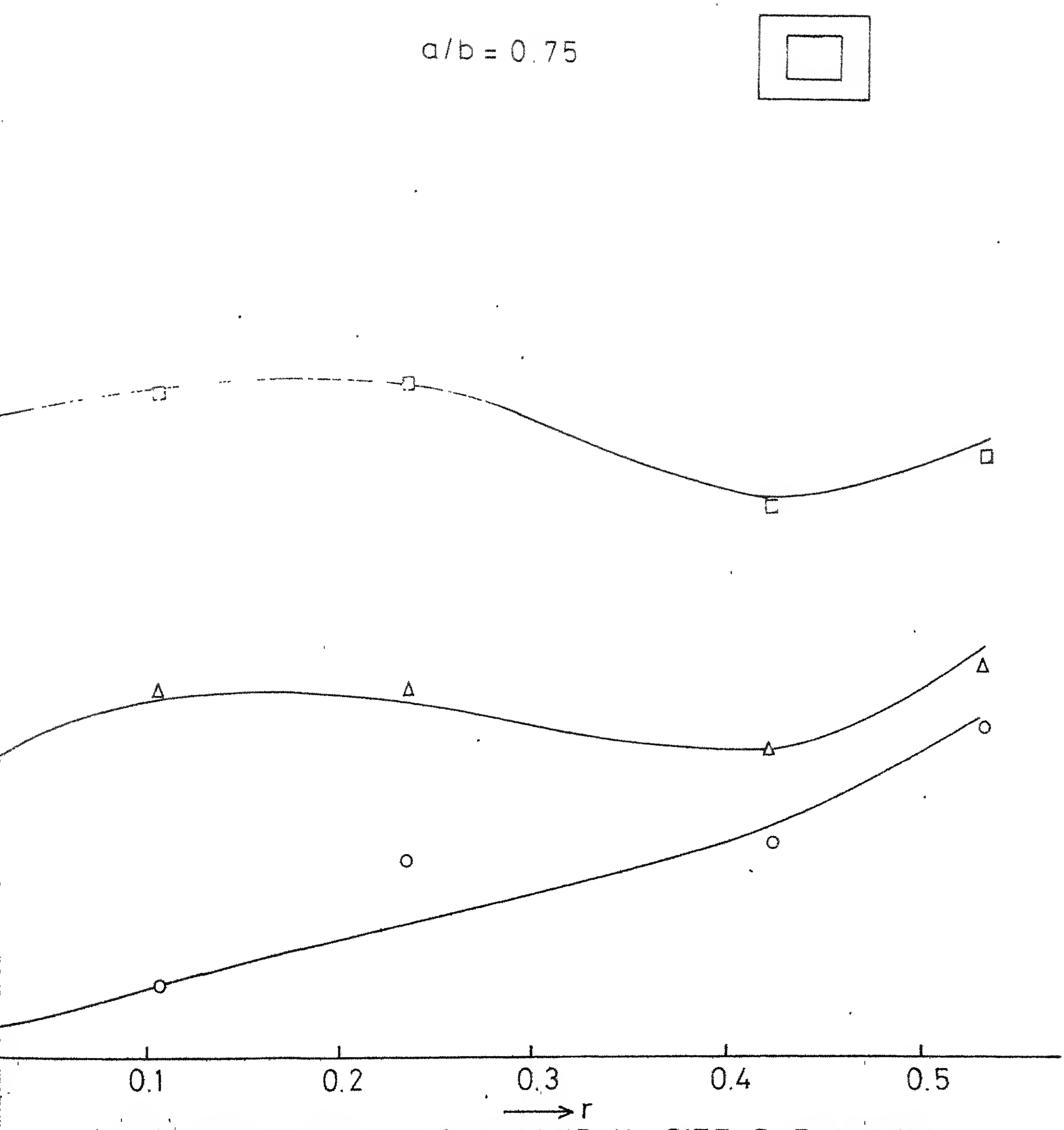
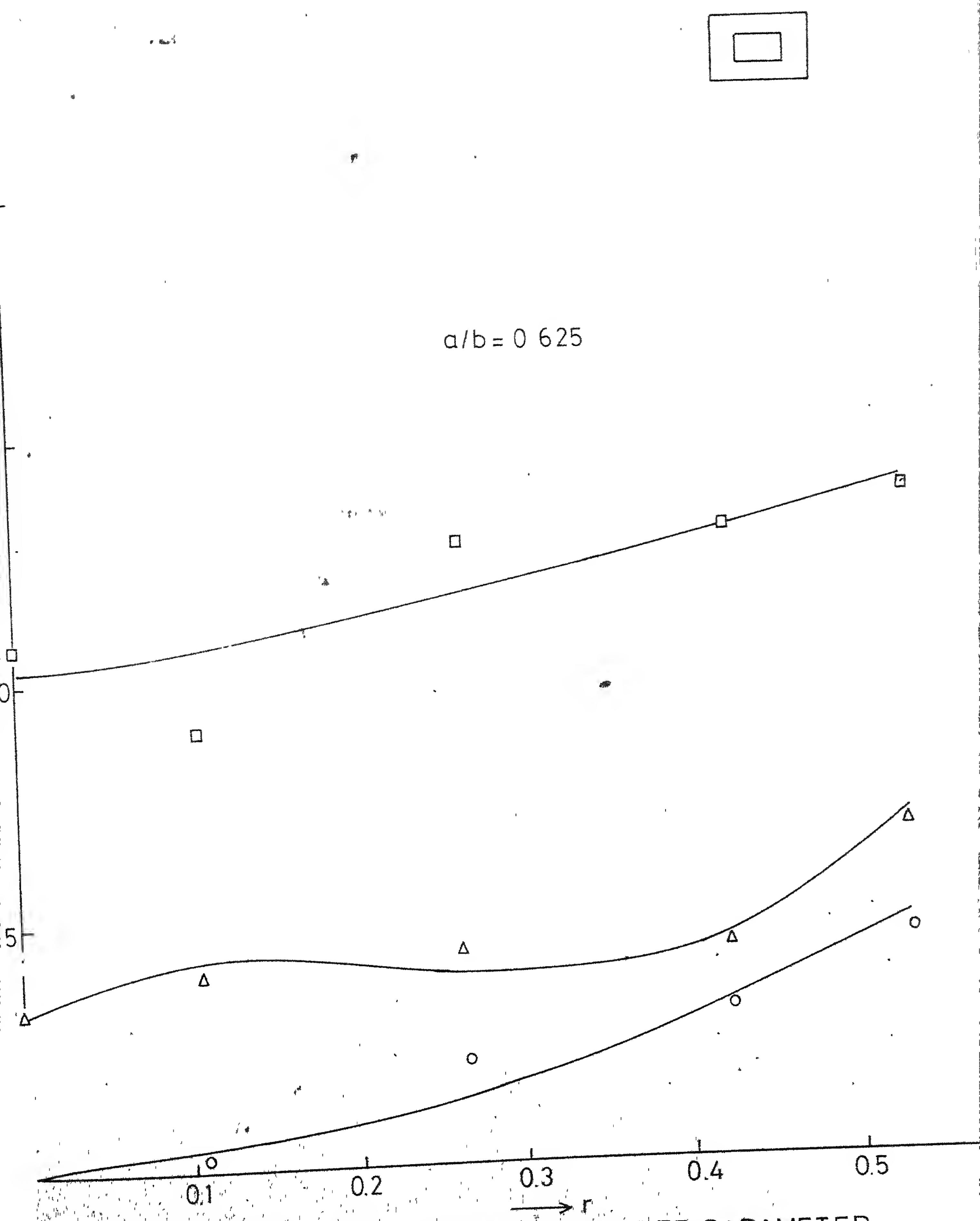
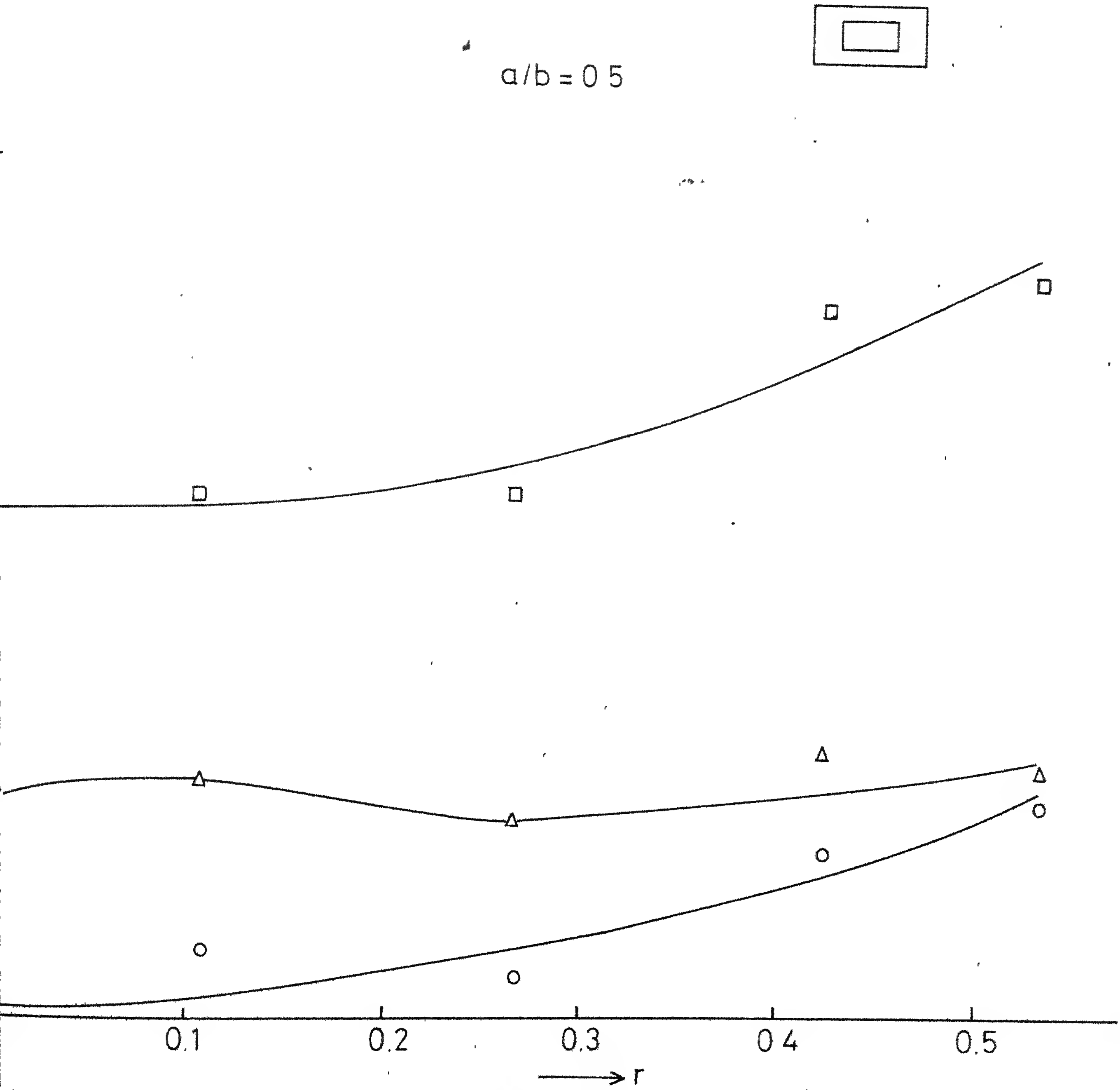


FREQUENCY RATIO PARAMETER Vs SIZE PARAMETER.



FREQUENCY RATIO PARAMETER Vs SIZE PARAMETER.

[redacted]



8. FREQUENCY RATIO PARAMETER Vs. SIZE PARAMETER

[redacted]

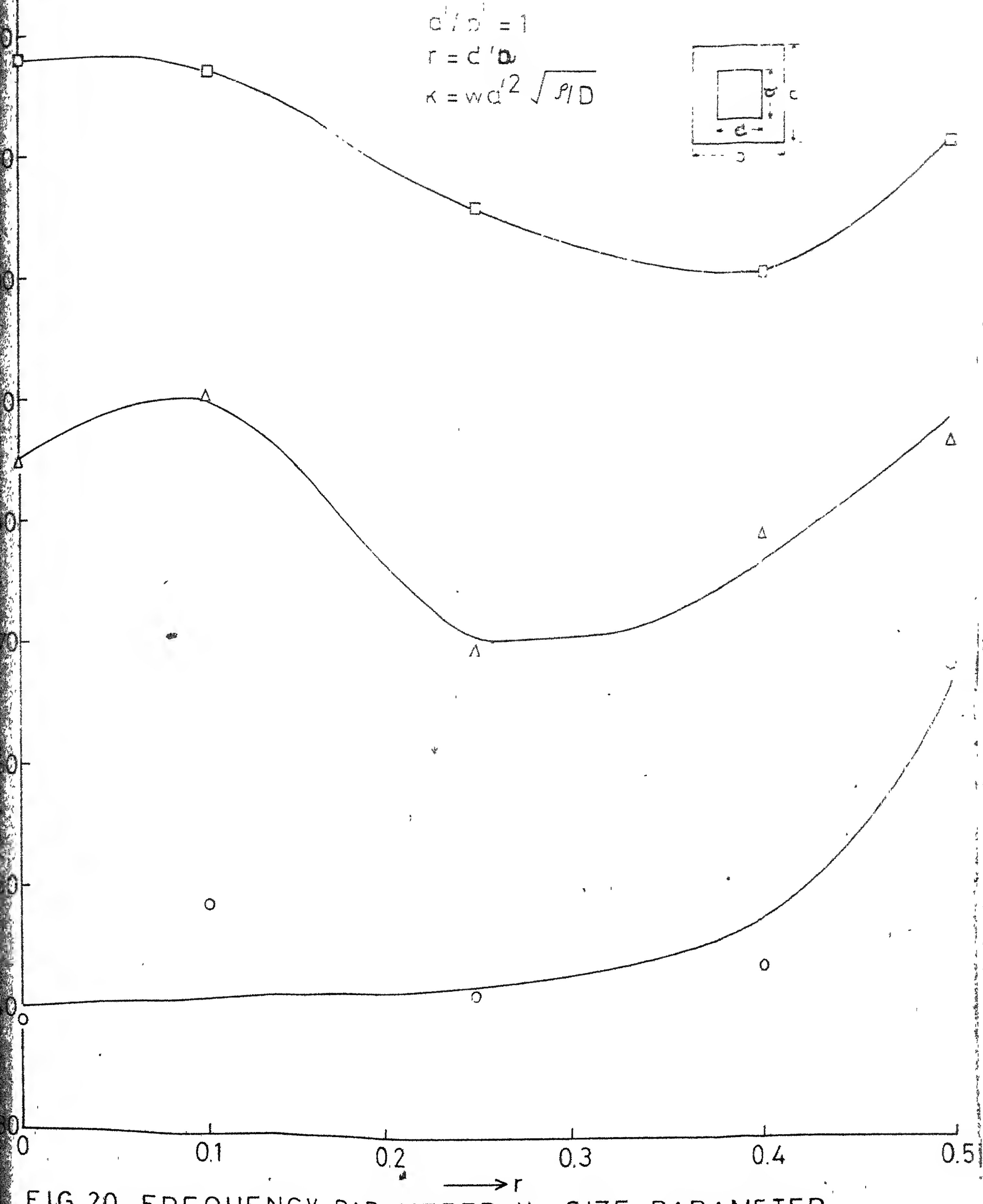
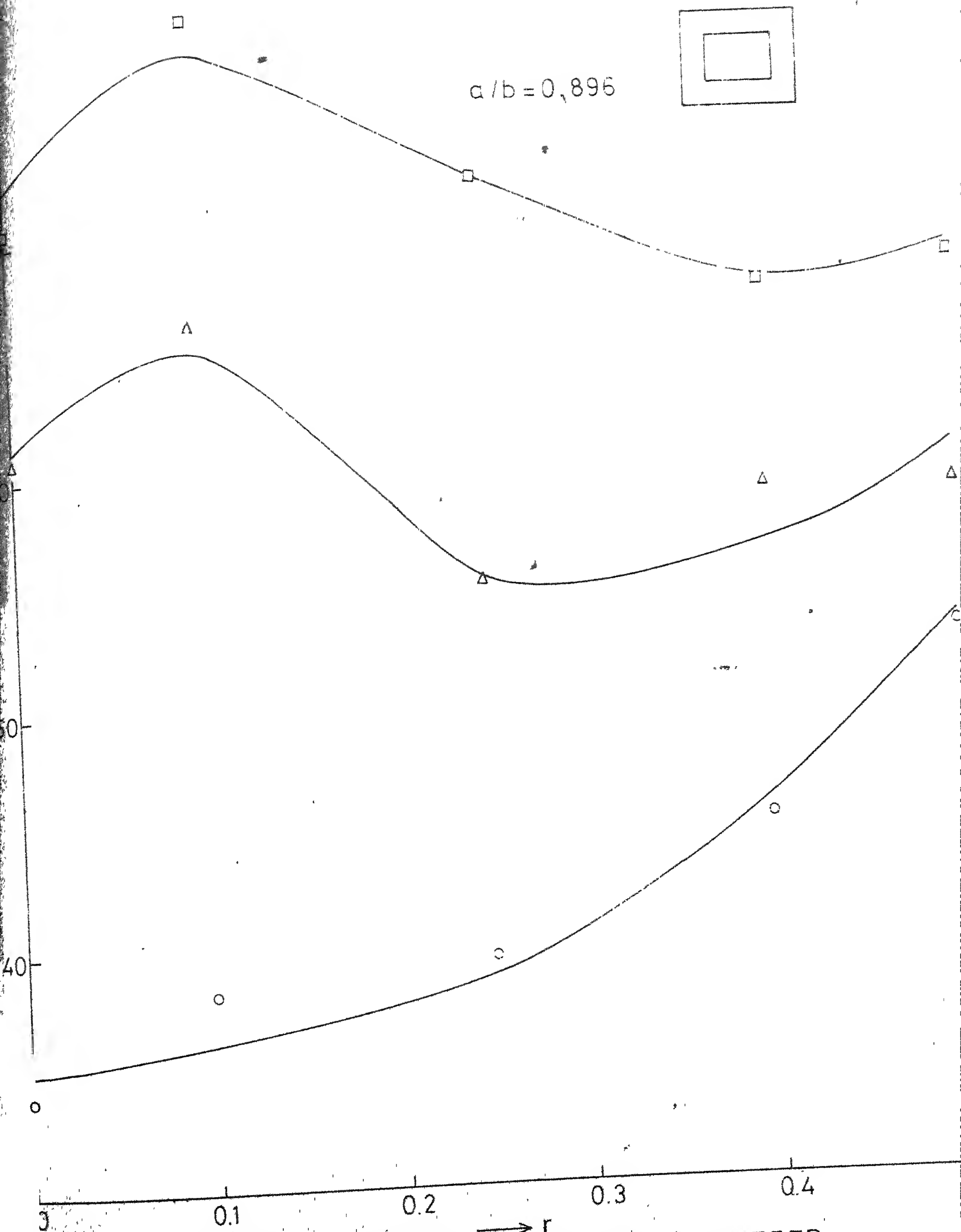


FIG. 20. FREQUENCY PARAMETER Vs SIZE PARAMETER.



FREQUENCY PARAMETER Vs SIZE PARAMETER.

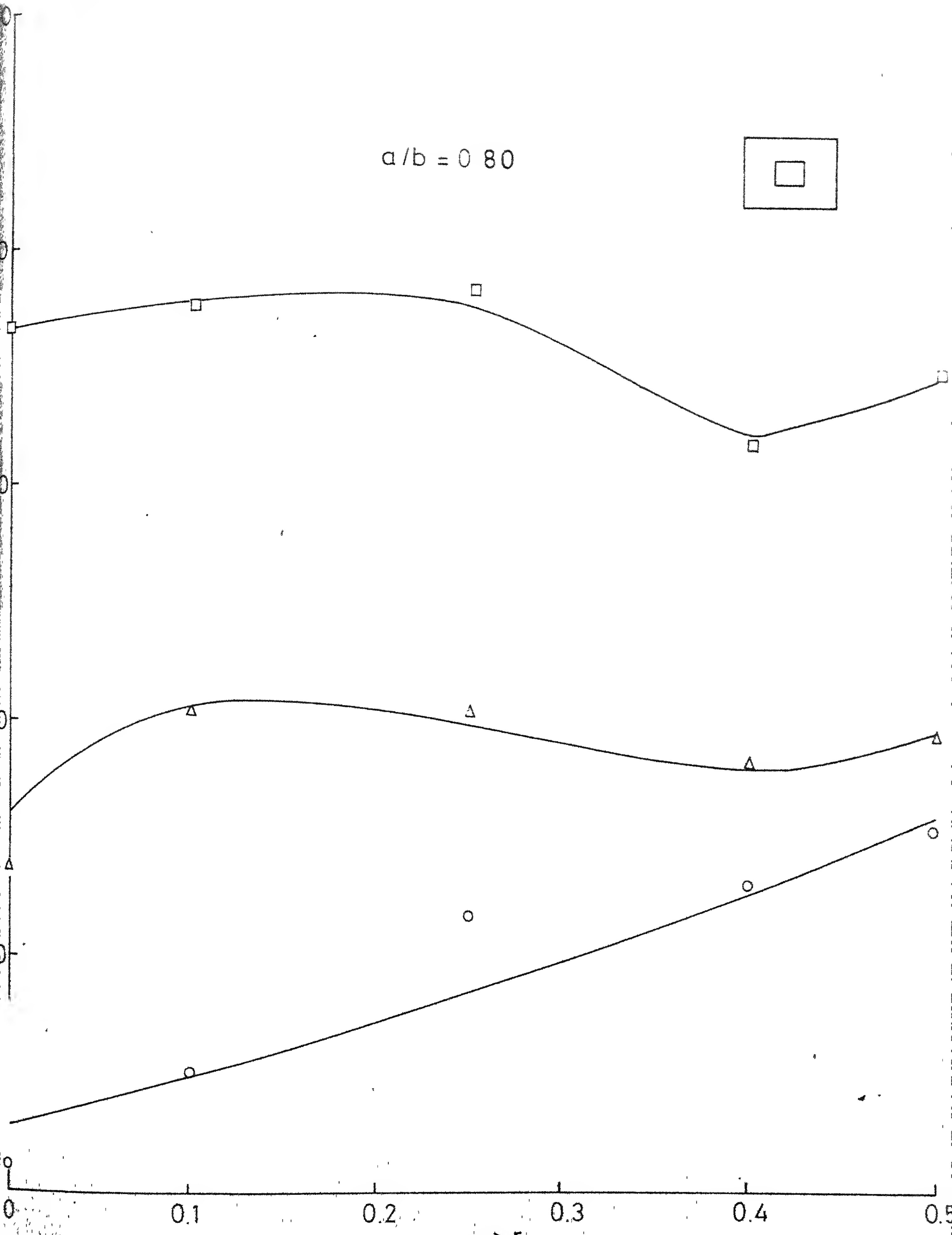


FIG.22. FREQUENCY PARAMETER Vs SIZE PARAMETER

[redacted]

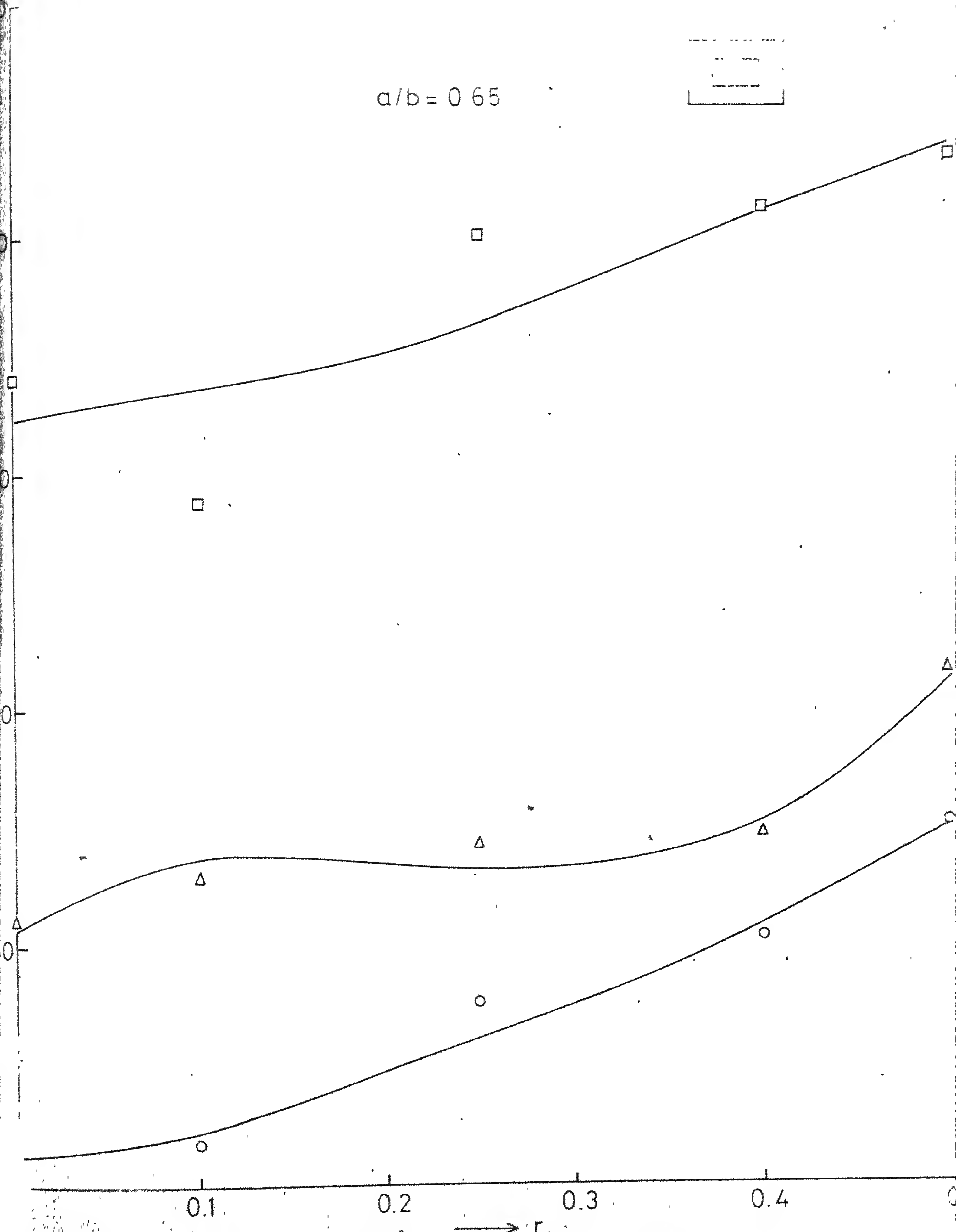


FIG 23 FREQUENCY PARAMETER Vs SIZE PARAMETER.

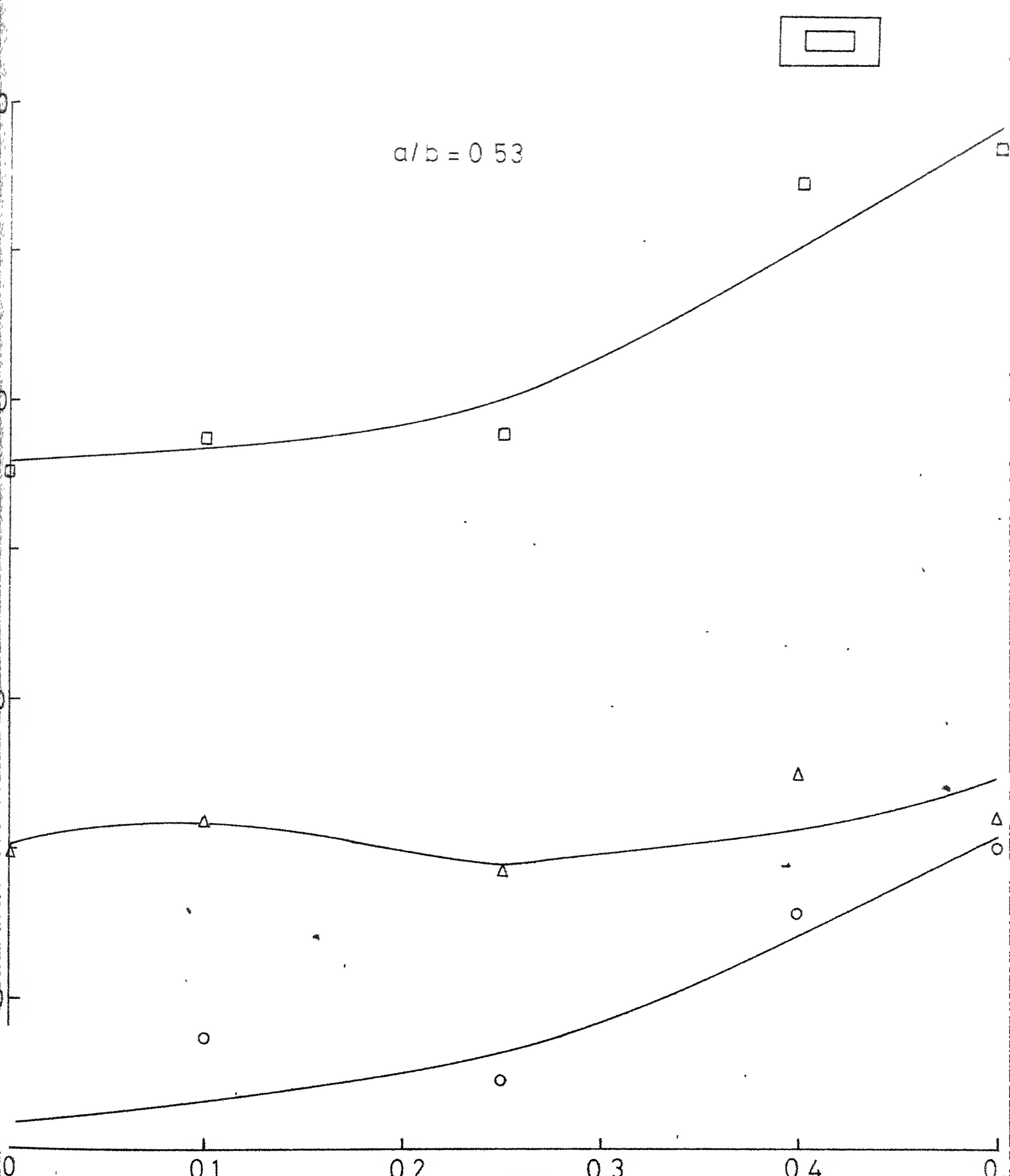


FIG. 24. FREQUENCY PARAMETER Vs SIZE PARAMETER.

TABLE 1 (a)

DETAILS OF PLATES TESTED

Thickness = 0.0364 in *

$E = 10.3 \times 10^{6}$ psi, $\rho = 0.098$ lb/in^{3}

No	Size of Plate (a x b)	Cut-out size parameter r
1	6" x 8" **	-
2	6" x 12"	-
3	6" x 12"	0.108
4	6" x 12"	0.266
5	6" x 12"	0.425
6	6" x 12"	0.534
7	7.5" x 12"	-
8	7.5" x 12"	0.108
9	7.5" x 12"	0.266
10	7.5" x 12"	0.425
11	7.5" x 12"	0.534
12	9" x 12"	-
13	9" x 12"	0.108
14	9" x 12"	0.266
15	9" x 12"	0.425
16	9" x 12"	0.534

Continued

[redacted]

TABLE 1 (a) Continued

No	Size of Plate (a x b)	Cut-out size parameter r
17	12"x 13.5"	
18	12"x 13.5"	0.108
19	12"x 13.5"	0.266
20	12"x 13.5"	0.425
21	12"x 13.5"	0.534
22	12"x 12"	-
23	12"x 12"	0.108
24	12"x 12"	0.266
25	12"x 12"	0.425
26	12"x 12"	0.534
27	9"x 12"	0.425 (eccentricity $\bar{y}$ = 1.25")
28	9"x 12"	0.425 (eccentricity $\bar{y}$ = 2.5")

* Thickness is kept constant for all plates tested.

** Cantilever plate.

[redacted]

TABLE 1 (b)

DETAILS OF PLATES TESTED

Area ratio Constant Z = 0.18

No.	Plate Size (a x)	Type of Cutout
29	9" x 12"	Circular
30	9" x 12"	Square
31	9" x 12"	Diamond (Rectangular)
32	9" x 12"	Rectangular

TABLE 2

NATURAL FREQUENCIES OF A RECTANGULAR CANTILEVER PLATE
(6" x 8")

Mode	1	2	3	4	5	6
Frequency cps	30	61	158	192	239	376

[redacted]

TABLE 3

NATURAL FREQUENCIES OF CLAMPLED - CLAMPED PLATES
WITHOUT CUTOUTS

Aspect ratio	Frequency, cps, for mode -					
	1	2	3	4	5	6
0.5	196	267	366			
0.625	160	212	331	374		
0.75	115	162	247	270	312	
0.89	74	132	153	203	237	281
1.00	85	185	256	301		

TABLE 4

NATURAL FREQUENCIES OF RECTANGULAR PLATES WITH CUTOUTS

$\bar{\text{A}}$spect Ratio = 1.0*

Cutout size parameter r	Frequency, cps, for Mode -		
	1	2	3
0.108	106	196	254
0.266	91	152	232
0.425	96	173	219
0.534	150	189	243

* is defined as width to length ratio.

[redacted]

TABLE 5

NATURAL FREQUENCIES OF RECTANGULAR PLATES WITH CUTOUTS

Aspect Ratio = 0.89

Cutout size parameter r	Frequency, cps, for Mode –		
	1	2	3
0.108	83	144	172
0.266	86	120	157
0.425	98	128	147
0.534	115	128	149

TABLE 6

NATURAL FREQUENCIES OF RECTANGULAR PLATES WITH CUTOUTS

Aspect Ratio = 0.75

Cutout size parameter r	Frequency, cps, for Mode –		
	1	2	3
0.108	130	190	250
0.266	155	190	253
0.425	160	179	229
0.534	183	194	241

[redacted]

TABLE 7

NATURAL FREQUENCIES OF RECTANGULAR PLATES WITH CUTOUTS

Aspect Ratio = 0.625

Cutout Size parameter r	Frequency, cps, for Mode -		
	1	2	3
0.108	164	221	303
0.266	194	232	364*
0.425	209	231	368**
0.534	234	268	380***

Frequencies observed between Modes 2 and 3

* 304 cps

** 286 cps

*** 341 cps

TABLE 8

NATURAL FREQUENCIES OF RECTANGULAR PLATES WITH CUTOUTS

Aspect Ratio = 0.5

Cutout Size parameter r	Frequency, cps, for Mode -		
	1	2	3
0.108	213	277	374
0.266	210	264	375
0.425	253	288	441*
0.534	269	278	449

* Resonance also occured at 302 cps

[redacted]

TABLE 9

NATURAL FREQUENCIES OF RECTANGULAR PLATES WITH ECCENTRIC CUTOUTS

Aspect Ratio = 0.75

Area Ratio Z = 0.18

Eccentricity of Cutout	Frequency, cps, for Mode -		
	1	2	3
0	160	179	229
$\dot{y}$ = 1.25"	127	183	252
$\bar{y}$ = 2.5"	114	170	263

TABLE 10

NATURAL FREQUENCIES OF RECTANGULAR PLATES WITH DIFFERENT SHAPE OF CUTOUTS

Aspect Ratio = 0.75

Area Ratio Constant, Z = 0.18

Shape of Cutout	Frequency, cps, for Mode -		
	1	2	3
□	162	176	280
○	165	181	265
▭	168	190	247
◇	161	179	250

[redacted]

TABLE 11

NATURAL FREQUENCIES OF RECTANGULAR PLATES FOR VARIOUS TORQUES APPLIED TO THE BOLTS.

Aspect Ratio = 0.5

	Frequency cps	Torque Applied		
		5 ft-lb	10 ft-lb	15 ft-lb
PLATE 1	Fundamental	192	197	196
	Second	244	266	267
	Third	359	368	366
PLATE 2	Fundamental	188	196	198
	Second	253	258	260
	Third	343	350	352

TABLE 12

COMPARISION OF EXPERIMENTAL AND THEORETICAL[10] RESULTS FOR A CANTILEVER PLATE (6" x 8")

Mode	1	2	3	4	5	6
Experimental frequency, cps	30	61	158	192	239	376
Theoretical frequency, cps	34.6	69.2	171	220	270	419
Difference %	15.3	13.4	8.2	14.6	13.0	11.4

[redacted]

TABLE 13 (a)

COMPARISION OF EXPERIMENTAL AND THEORETICAL RESULTS[10]
OF CANTILEVER PLATE (6" x 8")

n = 7

Mode	1	2	3	4	5	6
Experimental frequency, cps	30	61	158	192	239	376
Theoretical frequency, cps	30	63	161	192.5	242.5	380

TABLE 13 (b)

COMPARISION OF EXPERIMENTAL AND THEORETICAL RESULTS[10]
OF RECTANGULAR PLATE WITHOUT CUTOUT

Aspect Ratio = 0.89

n = 7

Mode	1	2	3	4	5	6
Experimental frequency, cps	74	132	153	203	237	281
Theoretical frequency, cps	73	135	154	214	239	283

TABLE 14 (a)

COMPARISON OF EXPERIMENTAL AND THEORETICAL FREQUENCY PARAMETERS[4] K FOR FUNDAMENTAL FREQUENCY OF SQUARE PLATES WITH SQUARE CUTOUTS.

	Frequency Parameter K, for Mode			
Cutout Size parameter	0	1/6	1/3	1/2
Paramsivam	34.85	35.8	43.25	62.4
Present Work	39.2	40.6*	45.9*	69.1

* Values taken from figure 20.

TABLE 14 (b)

COMPARISON OF EXPERIMENTAL AND THEORETICAL FREQUENCY PARAMETERS[4] K FOR A SQUARE PLATE WITH SQUARE CUTOUT CUTOUT SIZE PARAMETER $r = 0.5$

	Frequency, cps, for Mode -			
	1	2	3	4
Paramshivam Mesh size 8 x 8	57.25	68.55	98.52	143.95
Present Work	69.2	87.2	112	148.5

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[redacted]

TABLE 14 (c)

THEORETICAL FUNDAMENTAL FREQUENCY PARAMETERS OF A SQUARE PLATE WITH SQUARE CUTOUT FOR DIFFERENT MESH SIZE.[4]

$r = 0.5$

Mesh Size	Frequency Parameter K
8 x 8	57.25
12 x 12	62.37
16 x 16	66.05

TABLE 14 (d)

COMPARISION OF EXPERIMENTAL AND THEORETICAL,[2] AND EXPERIMENTAL AND EXPERIMENTAL FREQUENCY PARAMETERS K FOR FUNDAMENTAL FREQUENCY OF Rect. PLATES WITH CUTOUTS

	Frequency Parameter			
Area Ratio Z	0	0.0157	0.04	0.0625
Theoretical[2]	24.65	25.25	26.4	27.15
Present Work	26.0	26.8	27.5	28.1

	Frequency Parameter
Area Ratio Z	0.0384
Experimental[2]	26.65
Present Work	27.4

TABLE 15

FREQUENCY PARAMETERS K OF RECTANGULAR PLATES WITHOUT CUTOUT

n = 7

Aspect Ratio	Frequency parameter K, for mode -					
	1	2	3	4	5	6
0.532	25.55	34.8	47.7			
0.650	31.0	41.1	64.1	72.5		
0.765	31.1	43.8	66.6	73.0	84.6	
0.896	34.15	60.85	70.62	93.6	109.31	129.53
1.0	39.15	85.2	118.0	138.8		

TABLE 16

FREQUENCY PARAMETERS K OF RECTANGULAR PLATES WITH CUTOUTS

n = 7

Aspect Ratio = 1

Cutout Size Parameter h	Frequency Parameter K, for Mode -		
	1	2	3
.1	48.9	90.4	117
.25	41.9	70.1	106.9
.4	44.2	79.7	100.9
.5	69.1	87.1	112

[redacted]

TABLE 17

FREQUENCY PARAMETERS K, OF RECTANGULAR PLATES WITH CUTOUTS

n = 7

Aspect Ratio = 0.896

Cutout Size Parameter r	Frequency Parameter K, for mode -		
	1	2	3
.1	38.25	66.4	79.3
.25	39.65	55.35	72.4
.4	45.16	59.0	67.75
.5	53.0	59.0	68.65

TABLE 18

FREQUENCY PARAMETERS K, OF RECTANGULAR PLATES WITH CUTOUTS

n = 7

Aspect Ratio = 0.804

Cutout Size Parameter r	Frequency parameter K, for mode -		
	1	2	3
.1	35.15	51.4	67.6
.25	41.9	51.4	68.4
.4	43.25	48.4	61.9
.5	46.8	50.8	64.9

[redacted]

TABLE 19

FREQUENCY PARAMETER K, OF RECTANGULAR PLATES WITH CUTOUTS

n = 7

Aspect Ratio = 0.65

Cutout Size Parameter r	Frequency Parameter K, for Mode -		
	1	2	3
.1	31.75	42.85	58.7
.25	37.75	44.95	70.5
.4	40.4	44.8	71.35
.5	45.35	51.9	73.6

TABLE 20

FREQUENCY PARAMETERS K, OF RECTANGULAR PLATES WITH CUTOUTS

n = 7

Aspect Ratio = 0.534

Cutout Size Parameter r	Frequency Parameter K, for Mode -		
	1	2	3
.1	27.75	36.05	48.7
.25	27.35	34.4	48.9
.4	32.95	37.5	57.4
.5	35.05	36.2	58.5

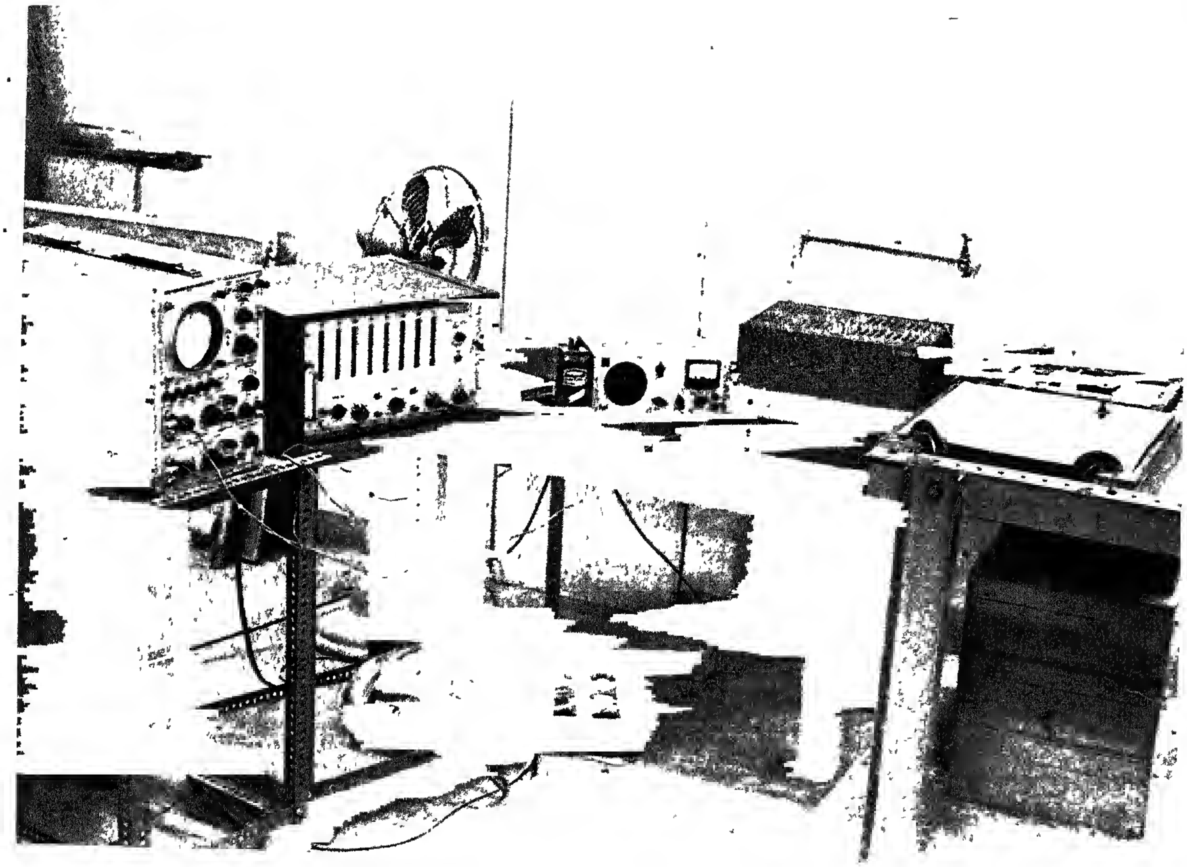

PLATE 1 : EXPERIMENTAL SET-UP

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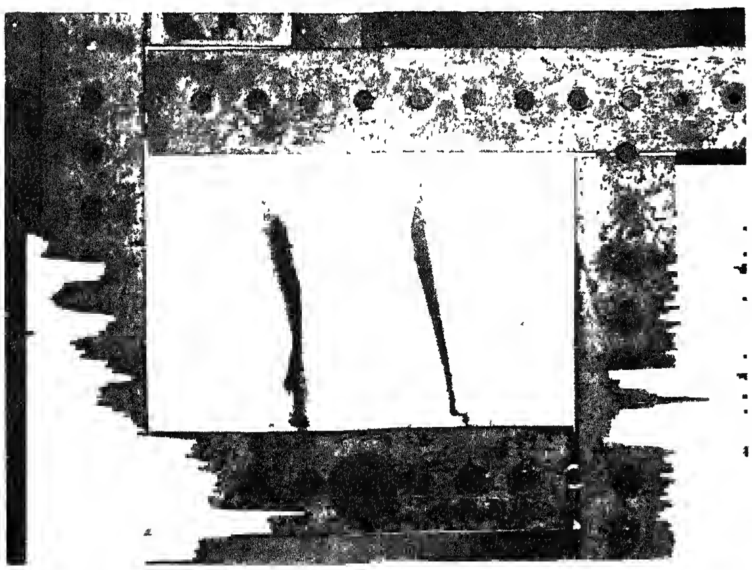

PLATE 2 : MODE SHAPE OF A RECTANGULAR PLATE
ASPECT RATIO = 0.625

[redacted]

CHAPTER IV

CONCLUSIONS AND SCOPE FOR FURTHER WORK

An experimental investigation was carried out to supplement the information on vibrations of plates with cut-outs. The experiments performed lead to the following conclusions.

(1) Good agreement between theoretical and experimental frequencies is obtained by taking $n = 7$ for effective length.

(2) The fundamental frequency always increases with increase in the size parameter r for all aspect ratios.

(3) For small values of r (upto 0.1) the second frequency first increases and then as r (from 0.25 to 0.35) is increased it decreases. But for r more than 0.35, the frequency again increases for all aspect ratios.

(4) In case of the third frequency, the behaviour of frequency parameter c depends upon both aspect ratio and the cut out size parameter. For aspect ratios (0.5 to 0.625) the frequency is found to increase continuously. For other aspect ratios (0.75 to 1.0) the variation of c with respect to r is the same as that obtained in second frequency.

(5) As the cut-out is shifted from the center towards clamped edges, the fundamental frequency decreases.

(6) For an area ratio of 0.18, the shape of the cut-out (diamond, square, rectangular, circular) does not affect the fundamental frequency.

Scope For Further Work

Since the actual conditions are neither clamped nor simply supported, the effect of the boundary conditions when the plate is partially clamped should be investigated. These results will be of use in the case of plates which are rivetted along the edges.

Effect of imperfections in the plates can be taken into account. Theoretical analysis for finding the natural frequencies of vibrations rectangular plates with rectangular cut-outs can be made.

Damping characteristics of plates with multiple cut-outs can be studied.

[redacted]

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